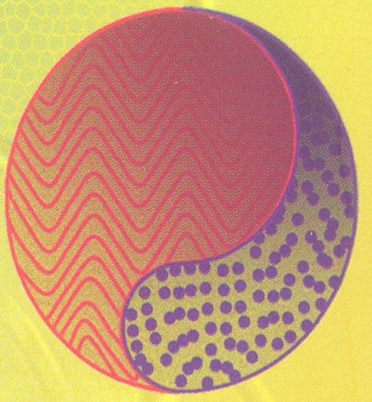
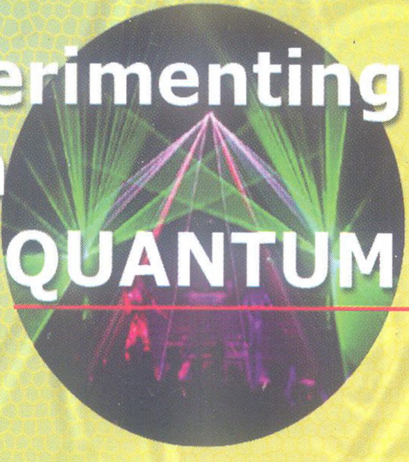
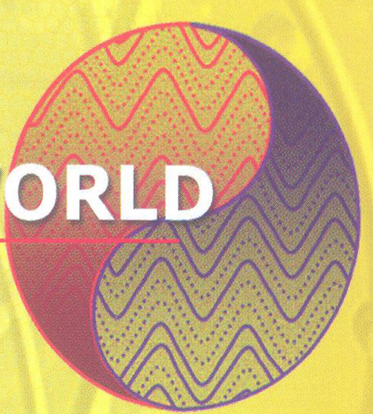
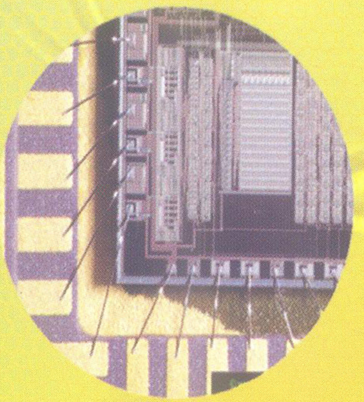
Experimenting with The QUANTUM WORLD









S T Lakshmikumar

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Vigyan Prasar

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नमो गुरुभ्यः

I am deeply indebted to everyone who helped me learn through teaching, discussion and argument.

Quantum mechanics fascinates a physicist. It is utilized to understand most physical phenomena and that too with incredible accuracy. At the same time, its assumptions are so counter- intuitive. Feynman said that it is delightful that we have to make such absurd assumptions in order to predict the results of experiments. While making qualitative description of experiments, most physicists have experienced the chagrin of forgetting the basic non-intuitive nature of the quantum objects such as electrons and making serious errors. Aside from this professional exposure to the quantum ideas and the difficulties in conveying the essence to the students, philosophical ramblings of non-physicists are a source of confusion and chaos. Feynman's small book "QED : The Strange Theory of Light and Matter" had shown me personally how to bring the depth and beauty of scientific understanding to the general public. Whatever little merit that may exist herein is due to this influence. I have little confidence that I have totally succeeded and would be most obliged if shortcomings and errors are brought to my notice..

I am indebted to Prof. Pankaj Sharan of Jamia Millia Islamia for a meticulous reading of the book and a series of corrections.. Dr S M Shivaprasad has always spared a lot of time for discussions and made many helpful suggestions for which I am grateful. I am also obliged to Dr Anurag Sharma, Dr P N Vijaya Kumar, Dr H K Singh and other colleagues at NPL

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Lastly and most importantly, I would like to thank Dr R Bhatttacharyya for his continuous encouragement and help during the writing of this book

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I
Introducing the Quantum Objects

To understand how x-rays are generated, how they are scattered by a crystal, how the atoms inside the crystal are organized and how the atoms have the specific properties, is impossible without quantum concepts. Mechanical properties such as strength and friction can only be understood in terms of the electrical forces of attraction and repulsion between atoms which are in turn purely quantum in nature. It is of course true that at the current level of understanding, the description may only be qualitative. For example, it is possible to understand the nature of bonding between carbon atoms quantum mechanically. Then it is obvious that graphite which has planar structure and weak inter-planar bonding must be a soft material. Diamond where the bonding is rigid in three dimensions is a hard material. It is however not possible to quantitatively calculate the strength of diamond and graphite for comparing with experimental results. Thus the description in this case is qualitative. Some phenomenon, such as colours of thin films of oil on water or the operation of the antenna of a radio receiver are explained using non-quantum mechanical theories (classical theories). However, as was beautifully described by Feynman in his beautiful book, "QED: The Strange Theory of Light and Matter" all these can also be understood on quantum basis. On the other hand, motion of cannon balls and planets, existence of the earth's atmosphere or the force required to slide a large box along the ground involve gravity and are not explained by quantum theories.

Quantum theories apply to a wide range of scales. They can be used to explain phenomena ranging in size from 10^{-18} m, the distances in an experiment studying the scattering of high energy particles, to 10^{4} m –

the length of the optical fiber in a quantum entanglement experiment which will be described later in this book. A quantum theory hypothesizes the existence of "quantum objects" which have many strange properties as will be described presently. It is not possible to form a mental image of the "quantum objects" with such strange properties. Thus quantum mechanics is described as being "counter-intuitive". However, experiments confirm this strangeness which is sometimes labeled as "quantum reality".

Non-physicists also have a deep curiosity to understand a concept as powerful as quantum physics. Books providing a description of quantum mechanics for the general audience are very common. Most of these books concentrate on the ultimate constituents of matter, the fundamental particles. Many try to link the "counter-intuitive" aspects of quantum mechanics to various religious and non-religious philosophies. There are also attempts to provide description of the peculiar experimental results obtained recently regarding "quantum reality" and the latest interpretations of quantum mechanics. As Feynman commented, in his book cited above, people want to know the "latest". The present effort seeks to explain, through a series of experimental studies, the strange properties of quantum objects that have been confirmed in a wide variety of physical phenomena and extensively used for technological development. At the end, it hopes to highlight the limited scope for interpretations and philosophies.

Unique nature of quantum explanations

Quantum explanations are extensively used in modern science. Quantum theory has predicted many new experimental results which were subsequently observed and the theory correlates mathematically a large number of experimental results. In certain experiments, the quantitative precision of the theory is superlative. Theoretical predictions match experimental results with an accuracy better than 1 part in 10^{10}. This is equivalent to theoretically predicting the experimentally determined diameter of the earth to the accuracy equivalent to the thickness of a human hair. In spite of this, the description that it provides is totally counter-intuitive.

The theory assumes that a beam of light consists of photons (light quanta). These behave in some experiments like particles localized in small region of space and time like a particle of sand. In other experiments the photon behaves like a wave of energy which is not localized (not confined to a small region of space and moving like a ripple on water). The smallest constituents of matter are once again quantum particles that behave like particles in some experiments and like waves in other experiments.

In addition to this dual nature, the theory claims that the properties of the quanta depend on the experiments performed to measure the properties. It predicts that experimentally correct theories cannot provide a description independent of measurement. It provides only a statistical description of nature. It enables a physicist to calculate the probability of the result of an experiment as an average over a large number of trials, but the result of a single trial cannot be predicted. It also insists that a theory which provides a non-statistical description will experimentally be shown to be wrong. Finally, quantum theory permits "spooky action at a distance" as Einstein put it. It permits a measurement performed at one location in space to influence the measurement at another location without the necessity of a signal traveling with a finite velocity to convey the information. Thus, quantum descriptions are the most accurate and universal scientific achievements but seem to violate commonsense (intuitive) ideas regarding reality.

Nature of quantum particles

While the brief comments made above would already be quite startling, a few more details are needed to describe a large number of experiments and understand their implications, as discussed in subsequent chapters. No mathematics is used in this book, in spite of the views of most physicists that a purely qualitative description is not very useful. However Feynman's QED, a book that has already been mentioned provides an example of what can be achieved without mathematics. The present description is an attempt to discuss many other applications of quantum theory in the same spirit.

Discreteness

The first attribute of a quantum description is, as the name itself suggests, discreteness. Quantum description uses the concept of localized entities which are called particles even if they are quite strange. Quantum description invokes the presence of quantum particles with mass which can be loosely termed as matter and quantum particles without mass which are called radiation, energy or even simply a photon. The energy content of the particles with mass (matter) can be changed due to the emission or absorption of other quanta. Thus, mass-energy equivalence is a general requirement of any quantum description.

Absence of superluminal (faster than velocity of light) signals

Mass-energy equivalence had been first deduced theoretically by Einstein as a corollary of his “special theory of relativity”. In simple language, this assumes that any change induced in an experimental setup, due to a change occurring far away, must reach the setup due to a physical cause. This physical cause cannot travel with infinite velocity. The limiting velocity is the velocity of light in free space. The special theory of relativity is a classical (not a quantum) theory. It describes classical objects, not quantum ones. In quantum theories also, no actual information can be sent at any velocity greater than that of light.

The two quantum families

The above restriction on velocity results in other interesting predictions of quantum theory. Firstly, existence of anti-particles is predicted. The particle and anti-particle annihilate (destroy each other), releasing the energy equivalent to the mass in the form of photons and other particles. Pauli’s spin statistics theorem is the second consequence. This theorem predicts that only two families of quanta can exist depending on a quantum mechanical property called “spin”. The “spin” is often visualized by describing a particle rotating or spinning about an axis. When describing a wave the “spin” is called polarization. For example, a light

wave is described as an electromagnetic wave with electric and magnetic fields. The direction of the electric field is its polarization. These are approximate classical descriptions. The spin of a quantum object is a multiple of a basic value h/2ð. This is called the quantum of action. "h" is called the Planck's constant in honour of Max Planck and is experimentally determined to be 6.626068×10^{-34} m^2 kg / s or $4.1356692 \times 10^{-15}$ eV. (One eV is the energy gained by an electron when accelerated by 1 Volt).

The first family of quanta are called Fermions. These have a "spin" which is an odd multiple of h/2ð. An ordinary object rotating in three dimensions returns to its original position after a rotation of 360º or 2ð radians. The fermion returns to the original position after a rotation of 4ð radians! The second family of quantum particles are called Bosons and are at least more conventional in returning to the original position after a rotation of 2ð. The spin value is an even multiple of h/2ð. The electron is the most famous fermion with a spin of ½. The photon is the representative boson and has a spin value of "1". Quantum mechanical spin also has a direction. This however has no value till it is actually measured.

Uncertainty

The most famous property of a quantum particle is the uncertainty in the measurement of one property for example the position of a particle, caused by the experimental determination of a second property, the velocity. Quantum theories insist that both the momentum and position of a particle cannot be determined simultaneously to unlimited accuracy. These pairs of quantities such as (momentum, position) and (energy, time) are called complementary variables. Another pair which is less well known is (number of photons, phase of the radiation). It is not prohibited to measure one of the pair of properties to any desired accuracy. However, the accuracy of the measurement of the second property will decrease. The product of the errors or uncertainties is limited by Planck's constant. In the quantum limit, conservation of energy can be violated over a time scale defined by the uncertainty relation. A particle with energy ÄE can be created, violating the conservation of energy for a time duration, Ät if the product ÄEÄt ≤ h/2ð. Such particles are known as virtual particles (excitations).

Entanglement

It is possible for two (or more) quantum particles to be in a peculiar state called entangled state. When the direction of the spin of one of the entangled particles is experimentally measured, it simultaneously and instantaneously defines the spin of the second particle. In principle, the two entangled particles can be separated by a distance of several light years. Measuring the property of one defines the property of the other instantaneously even though no signal can actually travel to the location of the second particle.

Decoherence

Notwithstanding the strangeness and counter-intuitive properties of quantum objects described above, quantum predictions invariably provide predictions that do not have these features at large sizes and energies. This change from counter-intuitive quantum properties to common sense is attributed to "decoherence", the loss of the quantum properties due to interaction between large number of particles and/or the environment.

Individual, composite and macroscopic quantum objects

The most interesting aspect of quantum theory is its wide applicability. Electrons and photons are not the only quantum entities. A proton behaves like a single quantum particle. A quantum theory of the nucleus however describes a proton as an agglomeration of three quarks. Considering neutrons and protons as quantum particles, an atom of helium consists of six fermions (two electrons, two protons and two neutrons). If the individual quarks are considered, there are fourteen fermions. The atom itself behaves as a single boson. All the quantum attributes such as uncertainty, wavelength, entanglement, decoherence etc. are experimentally observed for the helium atom. More complex examples of "quasi-particles" (objects formed by the interaction between two or more quantum particles) have been theoretically predicted and experimentally observed. The physics of condensed matter has thrown up a veritable zoo of quantum objects.

Even macroscopic objects displaying some quantum properties have been experimentally realized.

The plan

So far, a very brief introduction has been provided to the strange properties of quantum objects. The remaining chapters of the book seek to illuminate these ideas in detail. The second chapter, "Clues to quantum nature" attempts to identify the reasons for expecting a quantum description to be successful. While this to some extent is historical, many of the experiments presented are not the "classical" experiments used in physics text books. The ability of experiments to illuminate the idea is the criterion used for selecting them. Chapter III seeks to provide a descriptive account of the "collective behavior" of quantum particles. As quantum objects are extremely small, any macroscopic object can only be a collection of a very large number of quanta. The "collective behavior" is then described in terms of the basic fermion and boson properties. Chapter IV seeks to show the importance of quantum theory in technological development. A series of devices are introduced. These are devices which have been designed using the insights provided by quantum theory. Chapter V seeks to provide an up-to-date account of the experiments being performed, exploring the complex quantum mechanical ideas. This really reflects the strength of the quantum approaches. These latest complex experimental results are prospective candidates for technological applications and some of the future possibilities are described. The final short summary is a naive attempt in philosophy, emphasizing once again the breadth, depth and utility of quantum approach and pointing out the futility of attempting to escape from the counter-intuitive quantum ideas. It shows that quantum theory has to be accepted following the Holmes dictum: "When you have eliminated the impossible, whatever remains, however improbable, must be the truth."

II
Clues To Quantum Nature

As mentioned in the introduction, a quantum particle is sometimes assumed to be a particle to explain some physics experiments and as a wave for explaining others. Obviously, experimental demonstration of quantum nature is dependent on the performance of complex and precise experiments. The reason for the dawn of the quantum age with Planck's law in 1900 is the ability to perform precise experiments. In this chapter several experiments will be considered which will provide proofs of the quantum nature at the level of the individual quantum particle. Experiments demonstrating the wave and particle nature of the quantum objects, confirming the absence of superluminal communication of information, and those confirming that spin of a quantum particle has no value till it is experimentally measured are described. Entanglement, decoherence etc., are properties of multiple objects and will be considered in Chapter V. Not all the experiments cited here are used in conventional school text books. Some of them have been performed quite recently. The intention here is not to provide a historical description but to provide clues to enable the reader to immediately recognize the quantum nature of electrons and photons in these experimental observations.

Particles of matter

The atomistic philosophies of the Greek or Indian Philosophers are sometimes cited in science text books. However, it should be emphasized that these were completely irrelevant to the development of science. They were the result of imagination rather than experimental observation. The atomic theory as developed by Dalton, Avagadro and

other scientists of the 18th and 19th centuries are based on experimental observations. For example, the densities of various gases measured by Avogadro are shown in the table.

Table
Relative Density Measurements (Air = 1) of Gases by Avogadro

Molecule	Gas Density
Oxygen	1.10359
Hydrogen	0.07321
Nitrogen	0.96913
water vapor	0.625
nitric oxide	1.52092

The simplest explanation for the results is that water vapor and nitrous oxide are compound particles (molecules) formed by the combination of particles (atoms) of oxygen and nitrogen. The density of nitric oxide is equal to half the density of oxygen plus the density of nitrogen. Similarly water consists of two atoms of hydrogen and one of oxygen giving the text book formula of H_2O, since the density of water vapor is nearly equal to the sum of the density of hydrogen and half the density of oxygen. The high reactivity of nascent hydrogen corroborates these ideas by showing that gases like oxygen and hydrogen consist of molecules each of which are formed by combination of two atoms. Based on earlier quantitative measurements of the formation and dissociation of chemicals, Dalton proposed his atomic theory, whose main conclusions listed below are familiar to school students.

- Elements consist of tiny particles called atoms.
- All atoms of an element are identical and have the same mass.
- Atoms of different elements have different masses.
- Compounds consist of atoms of different elements combined together.

- Numbers of atoms of different elements combine in simple ratios.
- Chemical reactions involve the rearrangement of combinations of those atoms.

These rules are all based on precise quantitative experiments similar to those mentioned above and not on contemplation. For example, the issue of what constituted the space between atoms was a huge philosophical debate among the ancient philosophers. Such arguments are irrelevant for scientific atomic theory. The atoms and molecules are quantum objects. It is easier to observe the quantum properties of a particle smaller than an atom. Some experiments are however considered in subsequent chapters which demonstrate the wave nature of atoms and molecules.

Clues that particles smaller than the atom exist were provided in many early experiments. For example, Mendeleyev, the Russian scientist, who organized the elements into the periodic table, realized that the atomic weights of most elements are approximate multiples of the atomic weight of hydrogen. This is a clear clue to the existence of an internal structure of atoms. The observations of Faraday are more interesting. He determined that the same quantity of electricity that decomposed tin chloride into chlorine and 3.2 grains of tin also is capable of decomposing 0.49742 grains of water into hydrogen and oxygen. Please note the use of a non-standard unit of measurement, the grain. These are Faraday's original results. The very fact that it is possible to ionize the elements by removing the charge indicates the inner structure of atoms. While Faraday may not have actually done so, the law of electrolysis which showed that 96500 coulombs of charge deposit, one mole (Avogadro number), of any monovalent element would have given the first measurement of the charge of an electron. $96500/6.023\times10^{23} = 1.60\times10^{-19}$ C. Thus clues emerge in early scientific work on the possible existence of particles smaller than an atom of hydrogen, which has the smallest atomic weight. Almost at the same time as Planck's identification of photon, the electron with a mass of about 1/2000 that of a hydrogen atom was discovered. Its quantum nature will be demonstrated first.

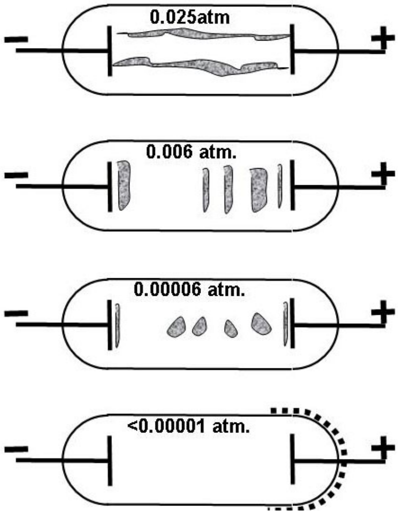

Figure II.1 Schematic of the basic discharge tube demonstrating the changes in coloured discharge patterns as the pressure is lowered until cathode rays cause a glow on the glass tube near the anode.

Electron: The smallest particle with mass

The electron with a mass of about 1/2000 the mass of a hydrogen atom is the smallest particle with mass. The existence of such a particle has been first confirmed experimentally by J. J. Thomson. He used equipment similar to the simple setup shown in Figure II.1, which is similar to the neon tube used now-a-days for advertisements. It is essentially a glass tube inside which two metal plates are located. Most of the air is removed by a suction pump. The metal plates are connected to a high voltage. As the air is sucked out of the tube with a mechanical pump, the pressure inside the tube is reduced and an electric current begins to flow between the electrodes. This is called an electric discharge which causes coloured light. The various colours needed for advertisement boards are obtained by changing the gas in the tube. As the pressure is further reduced, the light disappears and a weak glow is detected near the anode (The plate connected to the positive terminal of the power supply). For this

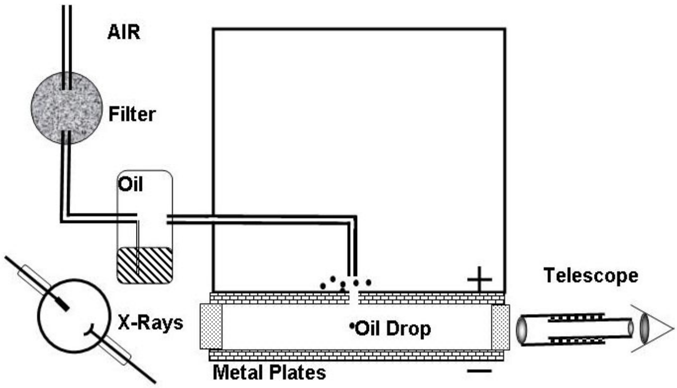

Figure II.2 Schematic view of the Millikan oil drop experiment for determining the charge of an electron.

glow to be visible, a small amount of phosphor material, similar to the coating in TV tubes, is placed in that region of the tube. The particles causing this glow can be moved using a magnet, showing that these are charged particles and they can also move a small paddle wheel confirming that they have momentum and thus a mass. The force on a charged particle, due to an electric field depends only on the charge while the force due to magnetic field depends on its charge, mass and velocity. Using this idea, J. J. Thomson determined the ratio of charge to mass. He used an electric field to deflect the particles from a particular point and used a magnetic field to bring them back to the same point. The two forces are now equal and the ratio of charge to mass of the particles could be determined. The experiments confirmed that the cathode rays, as they were called since they were emitted by the cathode and detected near the anode, consisted of negatively charged particles which were then named electrons.

Millikan's "oil drop experiment" is a very important next step. It determines the charge and mass of individual electrons with a high accuracy. A schematic view is shown in figure II.2. Very small oil drops are created by spraying oil in the chamber. A few of them drift down through the small hole to the space between the two metal plates and can be seen using a small optical microscope. The oil drops tend to settle down on the bottom

plate due to gravity. X–rays are used to ionize the oil drop by creating free electrons on the drop. An electric field is applied to the plates so that it attracts the drop towards the upper plate. This force is adjusted to balance the gravitational force so that the drop ideally becomes stationary in the chamber. Knowing the electric force and the viscosity of air, it is possible to determine the charge on the oil drop. It is then seen to be an integral multiple of the electronic charge 1.6×10^{-19} coulombs.

Electron as a wave

Wave phenomenon is an everyday experience. A stone dropped into still water in a pond or lake causes ripples to travel outward and the wave pattern can be easily seen. If two stones are dropped near one another, interference between the two sets of waves leads to formation of patterns like the one shown in Figure II.3. The most important aspects of wave motion are (1) energy travels along the wave but the material does not and (2) the interference between waves leads to some regions where there is no wave motion and others where there is a very large wave. If a small leaf is floating in this water it moves up and down with the wave, the

Figure II.3 Waves on water display constructive and destructive interference.

energy for this comes from the wave. At the same time, the leaf does not move in the direction of the wave. This is the experimental proof of the first statement. If the picture is observed closely, there are two sources of waves. There are also regions of flat water surface. If only one of the waves was present, there would be continuous wave motion in this region. The combination of two waves leads to regions where there is no wave motion. This is the experimental observation that supports the second statement. There are also regions where the motion of the medium due to the waves is enhanced. Thus there are both destructive (no motion of the medium) and constructive (enhanced motion of the medium) interference of waves. The tendency of one wave to move the particles of the medium in one direction are being exactly balanced by the tendency of the other source of waves to move the particle in the opposite direction causing the destructive interference. If both waves tend to move the particles of the medium in the same direction, the wave motion is increased. This is constructive interference.

Observation of interference confirms wave nature. The identification of the electron as a fundamental particle at the turn of the century was followed thirty years later by experimental proof of the wave nature of electrons. For interference, the separation between the two sources of waves must be comparable to the wavelength. When the electron is considered as a quantum object, it has a wavelength mathematically obtained by dividing Planck's constant h with the particle momentum (product of mass and velocity). This is called the de Broglie wavelength and decreases with increasing mass (h/mv). In view of the small value of the Planck's constant h, the de Broglie wavelength of even the smallest material particle will be quite small. Interference can only be observed when there are sources separated by such small dimensions. A crystal consists of atoms arranged in regular order separated by very small dimensions and can be used to create interference of electron waves. Davisson, Germer and G. P. Thomson (The son of J. J. Thomson who identified the particle nature of the electron) demonstrated that the electron is a wave with a wavelength given by $h/\sqrt{(2meV)}$. Here V is the voltage through which the electron of charge 'e' is accelerated. The schematic is

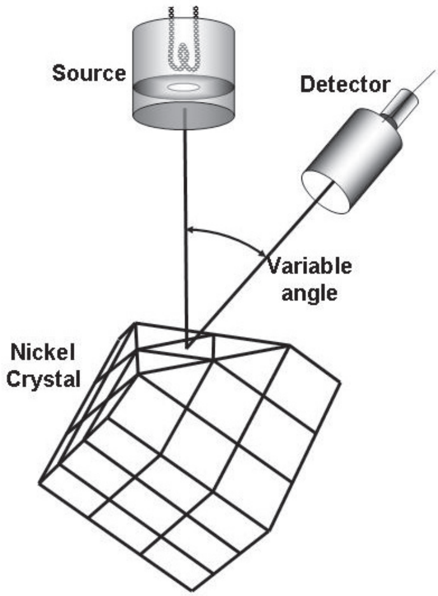

Figure II.4 Schematic of the Davisson and Germer setup which confirmed the wave nature of an electron.

shown in Figure II.4. The electrons emitted by a hot filament are accelerated by the application of a high positive voltage. They pass through a small aperture and a small beam of electrons falls on a nickel crystal. The number of electrons scattered in a given direction are determined for various angles and accelerating voltages. The results are shown in Figure II.5. The number of electrons detected depends on the angle. In some directions, a large number of electrons are detected demonstrating constructive interference. The regions of destructive interference correspond to directions along which no electrons are detected. The experiment however has to be performed in high vacuum to prevent the molecules of air interfering with the experiment. As shown in Figure II.5, in the experiment, electron waves scattered by two different rows of atoms are allowed to combine. These act as two sources and constructive interference which results when the distance between the sources d (the distance between nickel atoms in the crystal) and the wavelength ë obey the mathematical relation 2d sinè = në which is famous as Bragg's law. In the formula n is an integer and è is the

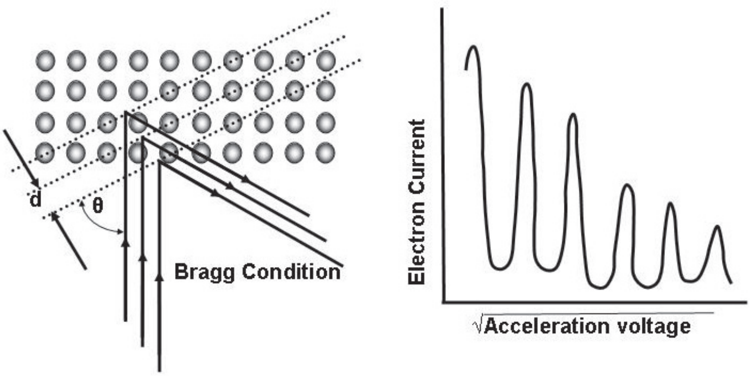

Figure II.5. The experimental results of the Davisson and Germer experiment show that more electrons are detected when the wavelength obeys the Bragg condition.

angle between the direction of the incident and detected electrons. Diffraction of electron waves, along with diffraction of X-rays are established techniques for identifying the ordered arrangement of atoms in solids. Basically the wave nature is assumed and the Bragg's law is used to calculate d, the distance between atoms in a crystal. The wave nature of particles larger than an electron is not easily observed since the wavelength decreases as the mass increases. The hydrogen atom is 2000 times heavier and thus its wavelength would be smaller by a factor of ~45. However, as will be discussed later, wave nature of these particles has also been experimentally confirmed in recent experiments.

Electron: Neither a particle nor a wave

A wave has been described as a propagation of energy in a medium. Destructive interference merely refers to the movement of the medium influenced by one wave being further influenced of the second. The wave particle duality is more perplexing than merely the fact that we describe an electron as waves and particles in different experiments. The key issue is to reconcile the destructive interference with a particle description. Feynman initially conceptualized a thought experiment involving the building up of an interference pattern with individual particles. After

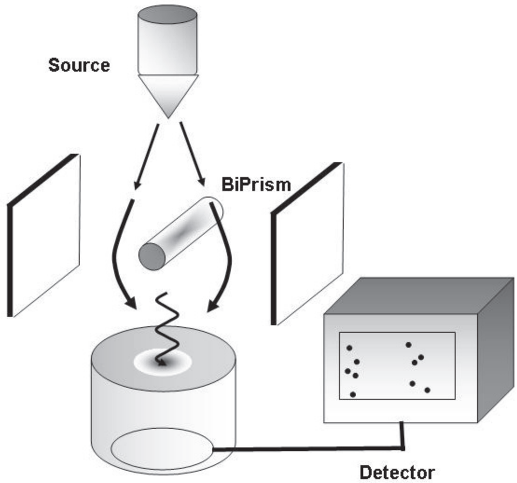

Figure II.6 Schematic of the experimental setup by Tonomura for demonstrating single electron interference.

sufficient numbers of particles are collected, one observes the interference fringes. This experiment has now been performed by Professor Tonomura and his group at Hitachi using a special electron microscope. The electron source is similar to the one used in the earlier experiments but the electron intensity is lowered so that electrons arrive one at a time. A thin wire is placed in the path of the electrons so that there are two possible paths (as shown in Figure II.6) for the electron, corresponding to the two sources of the waves. This experiment is possible because of the increase in sensitivity of the detectors that permits a single electron to be observed. As the electrons arrive at the screen one at a time, location is permanently recorded. When sufficient numbers of electrons are accumulated, there are very few electrons observed in some regions, which are the regions of destructive interference and large number of electrons in other regions (constructive interference). The slow accumulation of the data is represented in Figure II.7. Each electron randomly arrives at any location on the screen and no pattern is visible. In due course however, very few electrons are detected at some regions and the fringes become visible. This is one of the most beautiful of all quantum experiments and demonstrates the strange nature of the particles. An individual electron

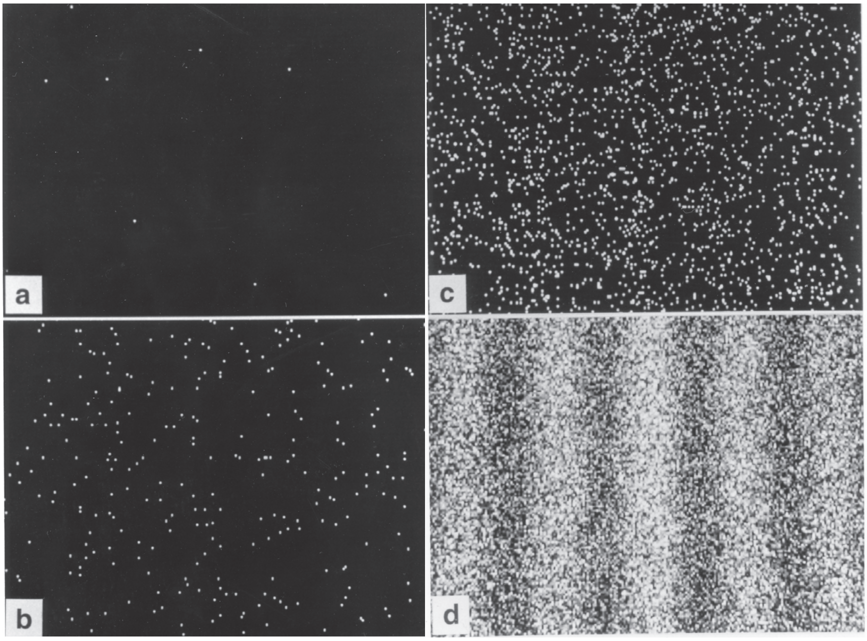

Figure II.7 Build-up of the electron interference fringes from (a) to (d) through accumulation of individual electrons. (Used with permission from Dr. Akira Tonomura)

cannot have any information about the possible location of the others. However, the statistical description is valid to a surprising degree. This experiment illustrates how quantum mechanics provides a statistical description while not predicting the single observation. Quantum mechanics encounters no problem. As Dirac stated, "the photon does not interfere with anything except itself". The probability of detecting an individual photon depends on the location. Two photons do not destroy each other unlike two waves. In the above experiment each electron has a very low probability of being observed in the regions of destructive interference and consequently the interference pattern is observed.

Light waves

If the wave phenomenon is accepted to be a means of transporting energy without any movement of material, the warmth experienced in

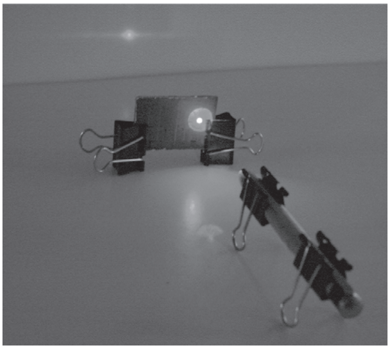

Figure II..8 Demonstration of interference with a laser pointer. (Used with permission from Dr. Stefen J. Edberg)

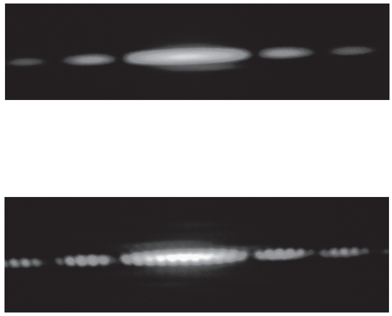

Figure II.9 Diffraction from a single slit and interference from two slits. (Used with permission from Dr. J. Newman)

sunlight should already be a guide to expecting light to be a wave. However, performing experiments that demonstrate the wave nature of light are difficult. The basic reason is the small wavelength. We now know that visible light consists of waves of length approximately half a micron (one millionth part of a meter). The first experimental proof of light waves was obtained by Young (the double slit experiment). He spilt a single source of light into two beams which were then recombined. The development of a laser has made simple demonstrations of the wave nature of light rather easy. A laser, despite their easy availability in our technological world, is not a simple light source (The distinction between ordinary and laser light sources will be discussed in the next chapter). The wave nature can be easily demonstrated by positioning a thin wire or a single hair perpendicular to the beam of a laser pointer when interference patterns become visible. In contrast, using a conventional light, the effect is very feeble and elaborate facilities are required to observe the patterns. The simplest experiment for demonstrating the wave nature of light is shown in Figure II.8. Light from a laser pointer falls on a mirror in which two narrow slits are scratched on the back (silvered) side. The patterns obtained on a distant wall or screen are shown in Figure II.9.

Another extremely simple result that can convince us of the wave nature of light is to observe diffraction or bending of light. Similar bending of water waves around obstacles and of sound waves around corners is part of everyday experience. Weak fringes can be observed by looking through two closely spaced (but not touching) fingers on a bright sunlit day. With care a couple of dark and bright bands can be seen along the edge of the finger. This is possible because the distance between the fingers can be controlled to a value comparable with the wavelength of light. The pattern obtained by diffraction of light due to a single slit is also shown in Figure II.9 along with the pattern due to interference from two slits. The substructure of interference pattern is due to diffraction at each of the single slits. The fringe width, (distance between successive dark regions) is mathematically related to the wavelength of light, the size of the slit and the distance to the screen. A historically important result is presented in Figure II.10. Light has been diffracted around a circular object such as a penny coin. The small spot of light visible at the center of the penny coin is not due to a hole in the object. It is the result of light bending around the edge of a circular object. It is called the Poisson Spot in honor of the scientist who mathematically predicted its existence and convinced the

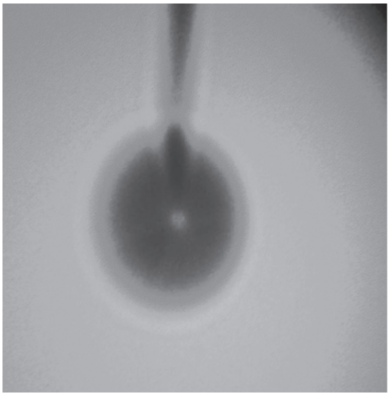

Figure II.10. The Poisson spot at the center of light diffracted around a circular object. (Used with permission from Dr. J. Newman)

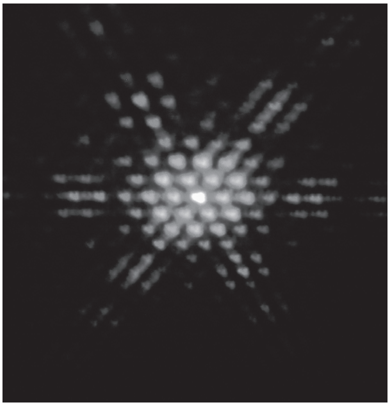

Figure II.11 Diffraction from a hexagonal aperture. (Used with permission from Dr. J. Newman)

scientific community of the wave nature of light. It should be emphasized that the Poisson Spot can be observed only for one value of the distance between the photographic plate and the circular object. Also the object has to be perfectly circular and the source of light placed exactly on the axis. One final result presented in Figure II.11 illustrates a very important aspect of diffraction. This is the pattern obtained from a hexagonal aperture. Clearly the diffracted pattern reflects the symmetry of the object that diffracts light. This illustrates the reason electron diffraction can be used to determine the order of real crystals as mentioned earlier.

Emission and absorption of light

The most common source of light is the sun. Sunlight, as Newton showed so dramatically, consists of a range of colours from the red to the blue. The various colours correspond to light waves of different wavelengths ranging from about 0.4 microns for blue light to about 0.7 microns for red light. Fraunhoffer observed that the spectrum of the sun (shown in Figure II.12 on colour plate 1) includes a large number of dark lines indicating the absence of that particular wavelength in the solar spectrum. In order to obtain a proper spectrum, a slit is placed between the spectrometer and sunlight. The sharp lines seen are actually images of this slit. Light of these wavelengths is absent in the spectrum of the sun. It is easy to identify the origin of these lines.If sodium vapor is heated to extremely high temperature, two sharp yellow lines are observed in the spectrum. The wavelength of the light emitted by sodium vapor at high temperatures corresponds to the two dark lines in solar spectrum in the yellow region. If the light from the hot sodium vapor is passed through cold sodium vapor, the two wavelengths are absorbed. Radiation from the center of the sun is composed basically of gamma rays. It becomes visible radiation after a large number of collisions along the way. Obviously, sodium atoms present in the outer surface of the sun have absorbed the radiation coming from the center resulting in the absence of these wavelengths in the solar spectrum. Interestingly, by noticing unknown lines in the solar spectrum, the element Helium ('from the sun' in Greek) was first identified. Just as with sodium, the spectra of copper and zinc shown in Figure II13 (in colour plate 1) consist of a number of sharp lines rather than a continuous spectrum. The

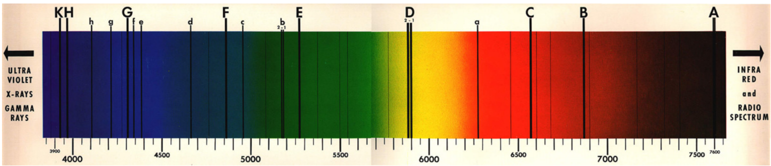

Figure II.12. The solar spectrum showing the dark Fraunhoffer lines.

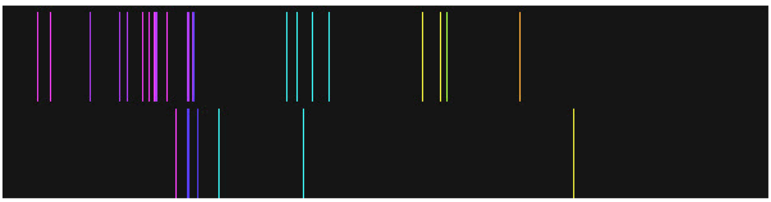

Figure II.13 Spectra emitted by copper and zinc at high temperatures.

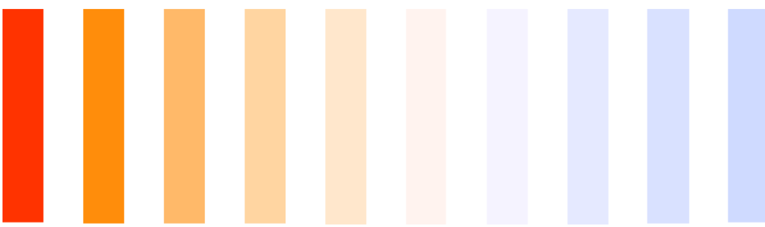

Figure II.14 The colour of a black body at temperatures from 1000 to 10,000 K (Steps of 1000K).

green lines in the spectrum show why the flame test performed for copper salts in the chemistry laboratory gives a characteristic green colour. The copper salt is heated to extremely high temperatures and the atoms release the energy as light of these colours. The comparison also shows that the lines are clearly characteristic of the element and not universal.

In contrast, the colour of most bodies is similar at very high temperatures. The colour depends only on the temperature, not on the material of the body. The radiation (light) emitted consists of a continuous range of wavelengths just like the solar spectrum. This is called the black body radiation. To observe the radiation accurately, a hollow body is selected and heated to the desired temperature. A small hole is made in the body and radiation coming out of the hole is measured. The variation of the perceived colour of a "black body" is shown in Figure II.14, (in colour plate 1) where the colour slowly changes from red to blue as the temperature is increased. The solar radiation (ignoring the dark lines) corresponds to black body radiation from an object at about 5,800 K. The energy emitted by a black body at various wavelengths is plotted in Figure II.15. As the temperature is varied, the shape does not change but the peak shifts toward smaller wavelengths. This accounts for the shift towards blue colour mentioned earlier. Precise experimental determination, particularly at lower wavelengths, of such spectra, was responsible for the

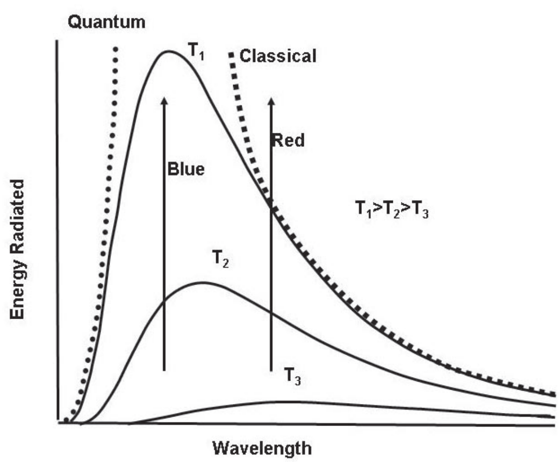

Figure II.15 Blackbody Radiation at different temperatures.

ad hoc quantization introduced by Planck. As Planck and later S. N. Bose discovered, this assumption leads to the derivation of a mathematical formula that agrees with the observations. Einstein, (in the paper popularly called the "paper on photoelectric effect" which incidentally was cited for his Nobel Prize Award) pointed out that Planck's radiation formula, which describes quantitatively the black body spectrum, represents light acting both as waves and as particles. If light was only a wave phenomenon the blackbody radiation should follow the curve marked classical in the Figure II.15. Just as the de Broglie relation relates the momentum (mass) of the particle and the wavelength of the associated wave, Planck's law (E=hí), relates the frequency of the wave of radiation to the energy of the associated particle (photon). In the formula, E is the energy, h is the Planck's constant and í is the frequency of the wave. It could be expected that a hotter body has higher energy and would emit particles of higher frequency (blue colour). If light was merely particles with the energy defined by Planck's law, the radiation should have followed the curved marked quantum. In a classical or wave description, the energy is not dependent on wavelength. A light quantum with long wavelength has a very small energy and consequently, the blackbody emits a large number of such photons. These collectively exhibit the classical behaviour at long wavelengths.

The broad energy spectrum of a hot body such as the sun can be contrasted to the sharp lines observed in the spectrum of atomic vapors. In order to explain the characteristic discrete sharp lines in the emission of a vapor, once again it is necessary to consider light to be both a particle and a wave. On one hand, light emitted, by a sodium vapour lamp for example, is a wave since it can be used as the source in the interference and diffraction experiments cited above. As mentioned above, the light from the same sodium vapour lamp, if analyzed, consists of two sharply defined wavelengths. The only way of accounting for the sharp lines in the spectrum is to use the famous Bohr's model of the hydrogen atom. Bohr assumed that the angular momentum of the electrons in the hydrogen atom is quantized. This ad hoc assumption can also be imagined to be a quantization of the length of the orbit (the circumference of an orbiting electron) being an integral multiple of electron wavelength. Consequently, the electron in the hydrogen atom (which is modeled as a miniature planetary

system with an electron revolving around the proton) can have only specific discrete energy values. The electron can exist only in successive levels each of fixed energy corresponding to the electron having specific wavelengths. The atoms absorb and emit radiation when the electron energy decreases or increases. The difference in energy is related to the wavelength of the light emitted by Planck's formula.

Photon as a particle

When metals are exposed to low wavelength light (typically ultraviolet radiation or X-rays), electrons are emitted. Einstein in the famous paper referred to above explained that the details of the process, number of electrons emitted, the dependence on the wavelength of light etc.can only be explained by a particle picture of light. However this analysis requires quite a bit of mathematical description.

The experiment performed by Brumberg and Vavilov is interesting and also very much simpler. The experiment performed in 1943 consists of a simple dark room in which a weak source of light, a light bulb operating at very low voltage was placed inside a box as shown in Figure II.16. The box had a small camera shutter which could be opened for small duration of time typically 0.1 sec. Since the observer is in a dark room his eye is adapted to darkness and is capable of detecting weak light. One would

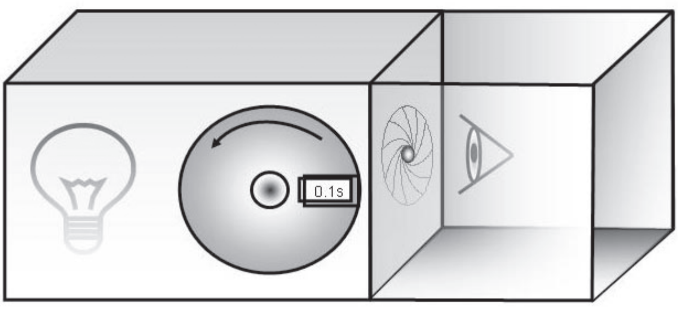

Figure II.16 Experimental setup of Vavilov and Brumberg consisting of a weak lamp observed in a dark room through a timed aperture.

expect that if the light coming out of the shutter was large enough to be detected, the eye would "observe" the light and if not, then the light would be "not observed". The results however are striking. At a given level of light intensity the eye sometimes detects the presence of light and some times it does not. It has to be clearly understood that this phenomenon is not a result of a mistake made by human perception. By considering light to consist of particles, the observations can be explained as follows.

The human eye has an ability to detect the presence of light when about 10-30 photons are incident on the retina in 0.1 sec. Since the light bulb is kept very dim, it emits a small number of photons. During the period of 0.1 sec while the shutter is open, if the required numbers of photons emerge from the aperture, they can be detected by the eye. If due to fluctuations, the number is smaller than the required limit, no observation of the light can be made. Aside from indicating that light consists of particles, these results provide an insight into the rate at which photons are emitted. In an ordinary light source, there is no reason for the emission of photons to be at a regular rate. Thus during a small interval of time, the number of photons emitted varies significantly. This tendency is called photon bunching. In the next chapter some light sources are introduced where the rate of emission of photons is stable, leading to interesting properties.

This experiment is remarkable in being the only simple experiment from which the quantum nature can be directly inferred. In most other cases a significant amount of mathematical analysis and precision experimentation is required. If the human eye happened to be an order of magnitude more sensitive, the nature of photons would have been self evident. But the human being would derive no benefit from this and as expected, the process of natural selection has not favored it.

Planck's law relates the energy of the quantum particle of radiation, the photon, to the frequency of the wave of radiation. The photon carries a specific momentum just like a material particle. The momentum of a photon is defined as the photon energy given by the Planck's formula

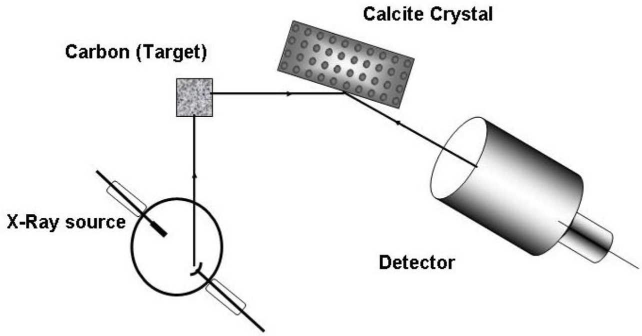

Figure II.17 Schematic arrangement of the experiment to demonstrate Compton Effect.

(E=hí) divided by the velocity of light (p=hí/c). The important experiment which confirms that photons have momentum is called Compton Effect.

The schematic is shown in Figure II.17. Compton performed the experiment with X-rays scattered from a target of carbon. The wavelength of the scattered radiation was in turn reflected from a crystal of calcite. Knowing the atomic distances in calcite, the wavelength of the scattered x-rays was determined. Radiation of wavelength ë is incident on an electron and the photon is reflected along a direction making an angle è. As in collisions between steel balls or marbles, the electron is deflected along another direction making an angle ö as shown in Figure II.18. As the electron moves in a new direction, its momentum is altered. The momentum

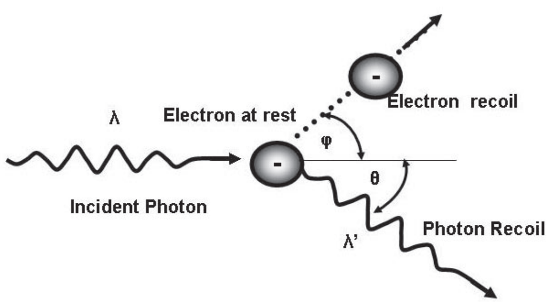

Figure II.18. Recoil of the electron at rest alters wavelength of incident photon.

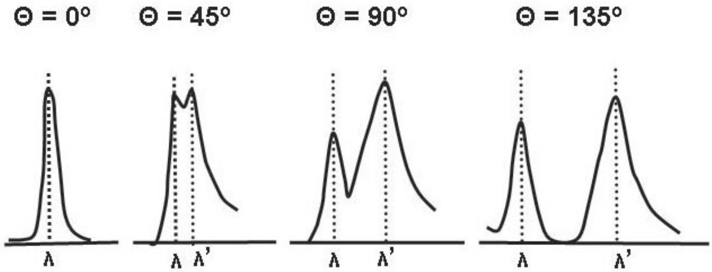

Figure II.19 Experimental results of Compton's experiment showing the variation of the wavelength with the angle of scattering.

is however conserved during the collision. Thus there is a change in the momentum of the photon. The momentum change of the photon results in change in the wavelength of the photon. Thus, depending on the angle at which the scattered photon is observed, it has a slightly different wavelength. The results are shown in Figure II.19. As the angle è is increased, the difference in energy between incident and reflected photon increases. Momentum transfer to the electron increases and as a consequence the difference between ë and ë' increases. This is exactly what one would expect from a collision of marbles. If the moving marble (photon) is reflected through a larger angle, the marble at rest (electron) would go further in the direction of the first marble which would also loose a lot of its original speed. This experiment confirms that the photons are particles which transfers momentum to the electron with which it collides. This experimental confirmation that light, in this case x-rays behave as particles had a large impact on the development of quantum ideas.

Photon : Neither a particle nor a wave

The experiments cited above illustrate situations when a photon behaves like a wave or as a particle. The ability to observe individual electrons has shown that these are classical approximations and that due to its quantum nature, an electron is beyond a simple description as a particle or a wave. The schematic of a similar experiment for demonstrating

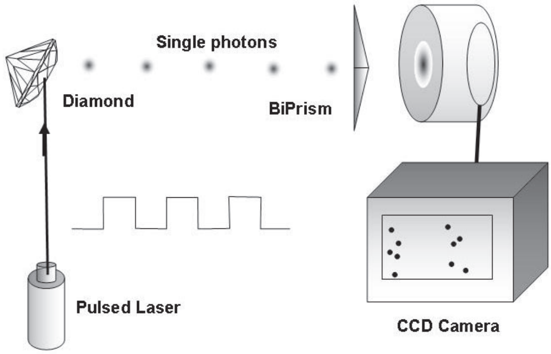

Figure II.20 Schematic of the experimental setup for single photon interference.

the quantum nature of a photon performed by a French group are shown in Figure 2.20. The experiment is performed using the single photon source which emits a single photon at any given time. These photons are emitted by a small 100 nm size diamond crystal which has a single nitrogen defect in it. When excited by a laser pulse, one photon is emitted per pulse. Since there is only one defect, there is no possibility of more than one photon being created with a single pulse. These individual photons are allowed to fall on a Fresnel biprism and the interference pattern is directly recorded by a CCD camera. Light falling on the two parts of the Fresnel biprism interfere just like light falling on the two slits in a double slit experiment. The final pattern obtained after 2000 s of the experiment shown in Figure II.21 are similar to the standard interference patterns shown earlier. However, the data collected at shorter intervals of time clearly shows that the individual bright bands are the result of a collection of large number of individual photons each of which appear as a single spot in the results obtained after 10 s. Clearly the formation of the final pattern is due to a statistical nature of the location of the individual spots. There is only a statistical probability for the photon to be observed in the location corresponding to the bright fringes. It is significant to note that while introducing the concept of a photon as a particle of light, (even while calling it a heuristic (trial and error) approach), Einstein in his very first publication makes a clear statement that the optical observations which are the basis for attributing wave nature to light refer to time averages and

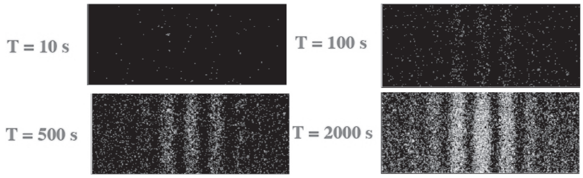

Figure II.21 Build-up of the photon interference fringes from (a) to (d) through accumulation of individual photons. (Reprinted with permission from V. Jacques, E.Wu, T. Toury, F. Treussart, A. Aspect, P. Grangier, and J.-F. Roch, The European Physical Journal D, 35, 561, 2005, © EDP Sciences, Societ'a Italiana di Fisica, Springer-Verlag 2005)

not to instantaneous values. A prophetic insight into the nature of light, notwithstanding his admission, shortly before his death, that he has not understood what a photon is. The later statement is only a rephrasing of Feynman's statement that nobody understands quantum mechanics. Actually no one can form a mental image of a quantum particle as these experiments prove.

Absence of superluminal signals

Both the existence of antiparticles and the classification of quantum particles into fermions and bosons are consequences of the special theory of relativity. In a simplistic description, the theory can be understood as a demand that any change induced in an experimental setup due to a change occurring far away must reach the setup due to a physical cause which necessarily cannot travel with infinite velocity. Experiments which demonstrate that the velocity of light is constant, irrespective of the motion of the source or detector, form the foundations of special theory of relativity. They are extremely important for quantum theory and can be claimed to be clues for quantum nature as well.

Mass-energy equivalence is an essential consequence of the special theory of relativity. It is necessary to retain conservation of energy during the quantum process of emission and absorption of photons. However, in the quantum limit, conservation of energy can be violated over a time

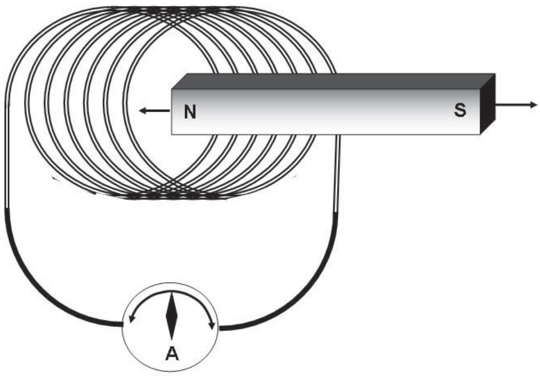

Figure II.22 The classic experiment of Faraday.

scale defined by the uncertainty relation. A virtual particle with energy ÄE can be created, for a time duration Ät if the product ÄEÄt ≤ h/2ð.

Einstein deduced the special theory of relativity from basic incompatibility of classical mechanics and classical electromagnetism. As the very first lines in his paper show, he was very much influenced by the peculiar situations with respect to the explanation of Faraday's experiment shown in Figure II.22. The magnet thrust into a coil causes a deflection in the galvanometer. Thus the electrons in the wire are charged particles moving under the influence of a force. If the observer is on the coil, the electrons in the coil are not moving towards the magnet but the magnet is moving towards the coil. An electric force is causing the movement of the electrons inside the coil perpendicular to the direction of movement of the magnet. The force is equal to the rate of change of magnetic flux. On the other hand, if the observer is on the magnet, the coil and with it the electrons inside the coil are moving towards the magnet. For the obsercver fixed on the magnet, the magnetic flux is constant. There is a force on the moving charged particle, perpendicular to the direction of motion due to the magnetic field (the Lorentz force). Thus the description of the cause for the movement of the electron depends on the location of the observer or on the "frame of reference" as it is technically called. What appears to be an electric field to a stationary observer is a combination of magnetic and electric fields for a moving observer. This explains why mere movement of

charge cannot cause emission of radiation (energy). Emission of radiation requires an acceleration of the charged particle. The physical insight of this equivalence enabled Einstein to show that the mathematical description of a physical theory must be Lorentz invariant (it must exhibit Poincare symmetry). The mathematical structure of quantum theory is Lorentz invariant. More mathematically, consider the movement of an object observed by two observers, who are moving with a constant velocity with respect to one another. They determine experimentally that the object has moved a distance $Äs_1$ and $Äs_2$ and for durations $Ät_1$ and $Ät_2$ respectively. These pairs are individually not equal but $Äs_1^2 - c^2Ät_1^2 = Äs_2^2 - c^2Ät_2^2$. This is the quantity that is conserved.

Michelson-Morley experiment is popular for demonstrating that the velocity of light is an absolute constant. This enables mathematical derivation but the conclusion is not really obvious to the student. The simple experiment shown schematically in Figure II.23 is much more satisfying. The experiment monitors the decay of the neutral meson ðº which is normally observed in cosmic rays in the upper atmosphere. The interaction of a $ð^-$ meson with a proton results in the production of a neutral meson ðº which decays as $ð^- + p$ ➔ $n + ðº$ ➔ $n + ã + ã$. The velocity of

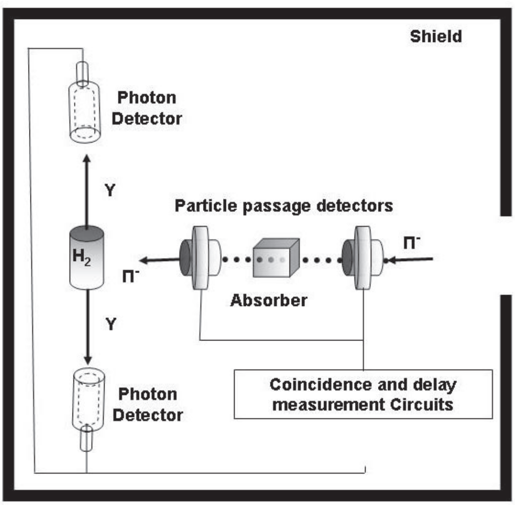

Figure II.23 The schematic of the gamma ray measurement setup for confirming the constancy of the velocity of light.

the neutral meson at the time of decay is ~0.2c. As shown in the schematic of the experiment in Figure II.23, if the velocity of the ã ray photon was dependent on the velocity of the meson which was 0.2c, then the two detectors placed at equal distances from the source should not detect the ã ray photons at the same time. The photon should take less time to reach one detector since it should be traveling with a velocity of 1.2c and more time with the other since it should be traveling with 0.8c. (in one direction the velocity of particle and light are added and subtracted in the other). Experimentally however, it is confirmed that the two photons arrive simultaneously. The velocity of the photon traveling in the direction of the neutral meson is equal to the one traveling in the reverse direction. This is a direct experimental confirmation that the velocity of light is a universal constant irrespective of the motion of the source and the detectors. If we consider a man swimming in the direction of and against the flow of a river, the normal rules of day to day life specify adding velocities. These do not apply for light. It should also be noticed that in this experiment, light is exhibiting particle nature.

The constancy of the velocity of light when the source is moving is much more simply appreciated by the absence of the ghost images of stars. Consider a binary star located in such a position that one of the stars is moving towards and the other away from the observer on the earth as

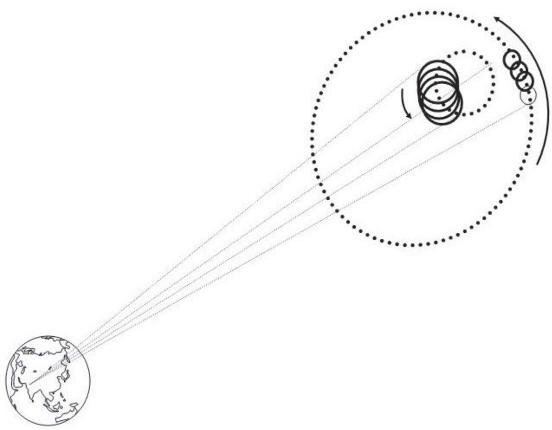

Figure II.24. The positions of the binary stars that should be visible simultaneously if the velocity of light were not constant.

depicted in Figure II.24. When the star moves towards the observer, if light is faster, it reaches along with the light emitted at an earlier instant. When the star moves away from observer, if light is slower it reaches along with light emitted later. Thus one should expect a series of ghost images of the star which are in fact never observed. While the argument is not considered to be absolute since there can be changes occurring to the state of the light as it travels from the star, the lack of ghost images is a very striking result and similar results have been obtained more accurately with orbiting satellites.

Einstein proved mathematically that the mass-energy equivalence is a consequence of the Lorentz invariance. The decay of the neutral meson mentioned above is an evidence for mass-energy equivalence. The experiments of Walton and Cockroft are historically credited with the first confirmation of the equivalence. In the experiment, bombardment of lithium metal with accelerated (700 keV) protons resulted in the reaction ${}_1H^1$ + ${}_3Li^7$ ➔ ${}_2He^4$ +${}_2He^4$. The mass in atomic mass units (amu) is 8.0241 for the Li and proton while it is 8.0056 amu for the two helium nuclei. The kinetic energy is 700 keV for the proton and 17 MeV for the two helium atoms. These are experimental values. It can be seen that the loss of mass, 0.0185 amu corresponds to 3.07×10^{-26} g or 27.6×10^{-6} ergs and is equal to the increase in kinetic energy of 17 MeV or 27.2×10^{-6} ergs.

Spin and polarization

Polarization is a property that can be easily visualized for a light wave. An electromagnetic wave is visualized as a propagation of time

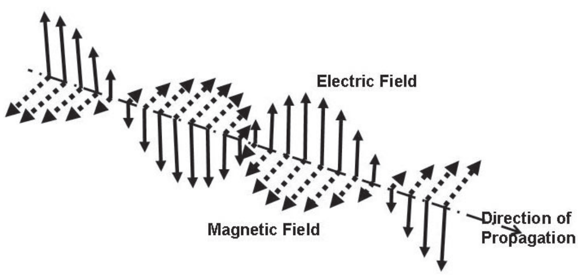

Figure II.25 The electromagnetic wave.

varying electric and magnetic fields (Figure II.25) which are mutually perpendicular and also perpendicular to the direction of propagation of the wave. It is possible to define the direction of the electric field as the direction of polarization. Thus, linearly polarized light has the electric field oriented in a given direction in space and un-polarized light has waves where the electric field is oriented arbitrarily all directions. It is also easy to visualize a polarizer as an object that would permit the motion of the electric field in one direction very much like a picket fence shown in figure II.26. When two polarizers are rotated at 90º to one another, it is not possible for light to pass through. On the other hand, when they are oriented in the same direction, light can pass through. In this wave picture, the introduction of a third polarizer oriented in a random direction between two crossed polarizers is easily described. The magnitude of the electric field that emerges from the polarizer slowly decreases as the angle between the two is increased. Mathematically this obeys a cosine law. The intensity of light transmitted is proportional to the cosine of the angle between the two polarizers as can be verified experimentally.

Since light is being described as a collection of photons, polarization has also to be described as a property of the particle. Since it is possible to experimentally detect a single photon, it is possible to investi-

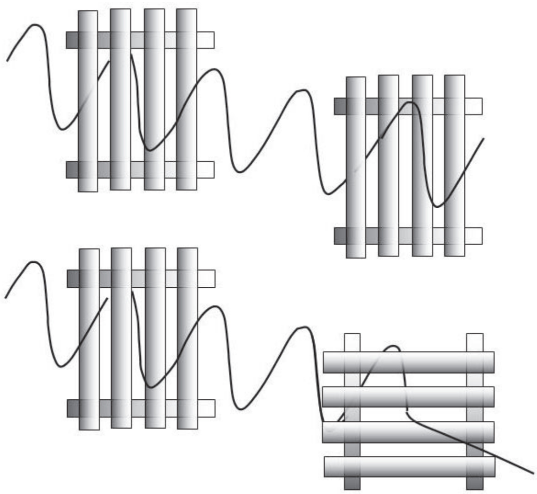

Figure II.26 A wave can pass only if fences are aligned.

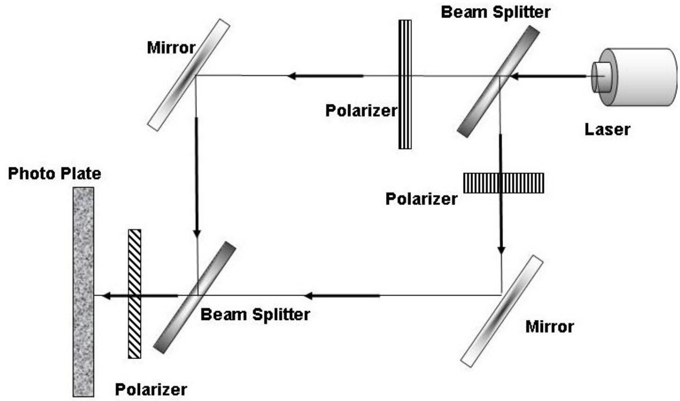

Figure II.27. The Mach - Zehnder Interferometer.

gate the polarization of a single photon. One experiment that evaluates the polarization of single photons is schematically shown in Figure II.27. This arrangement is called a Mach-Zehnder interferometer where the light from the laser is split into two parts at the first beam splitter and recombined in the second. The laser intensity is reduced so that there is only about 1 photon in about 100 s. This actually means that the photon counter detects about 30 photons per hour. Two polarizers, one oriented in the vertical direction and the other in the horizontal are placed in the two arms. Another polarizer is placed in the path of the combined beam. The simplest assumption regarding the polarization of a photon is that the photon has a specific direction of polarization. The photon passes through the vertically aligned polarizer if its polarization matches this orientation. Similarly for the horizontal polarizer. Thus, a photon incident on the third polarizer can be in either one of the two polarization states. If the third polarizer is placed at an angle of 45°, randomly, some photons from each of the arms should pass through leading to a uniform light intensity of the type shown in Figure II.28 (a). However, experimentally this is observed when the polarizer is removed. The interference patterns shown in Figure II.28 (b) and (c) are observed when the third polarizer is inclined at ±45°. Obviously the assumption that the photon has a specific polarization state associated with it is not correct. If the photon has a polarization state in a particular direction even before the experiment is performed, the photons

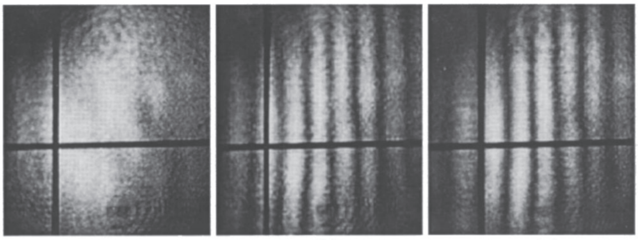

Figure II.28 The experimental determination of single photon polarization in the Mach- Zehnder interferometer. (Reprinted with permission from M. B. Schneider and I. A. LaPuma, American Journal Of Physics, 70, 262, 2002 © American Association of Physics Teachers 2002)

would have to be in one arm of the interferometer or the other leading to absence of the interference patterns when the third polarizer is at the given angle. This is contrary to observations. It turns out that the individual photon has a probability of a specific polarization direction but the actual direction cannot be specified till the experiment is completed. It becomes necessary to assume that the photon has no polarization state till it is actually measured. The classical result is however obtained for a sufficiently large number of photons.

The above experiment has confirmed that polarization of photon is a peculiar property that has no specific value till it is actually measured. It is relevant to look at the case of the material particles such as electrons. In the context of a particle, the polarization is referred to as "spin". The spin of an electron is often described as the spinning of a small sphere about an axis. The experiment has not been performed so far using electrons. The experiments of Stern and Gerlach are used to show the equivalence between the spin of the electron and the polarization of the photon. The experiment was performed by passing a beam of silver atoms through an inhomogeneous magnetic field. In terms of spin, the neutral atom of silver behaves exactly as an electron is expected to behave. If the silver atom is a classical small sphere spinning about an axis and having a small magnetic moment associated with it, the direction of this axis can be

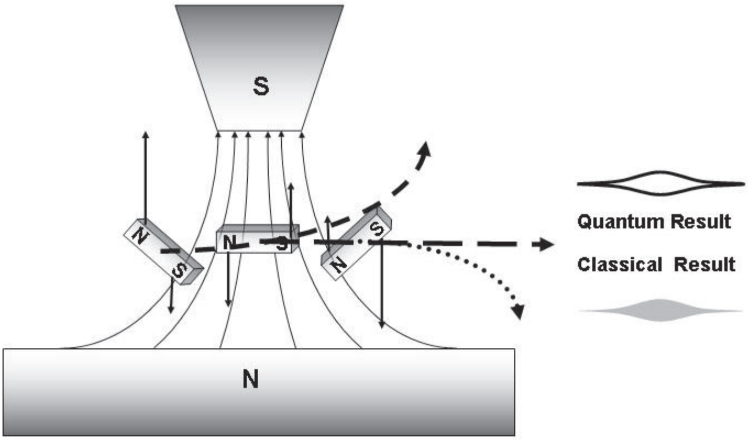

Figure II.29 Schematic of the Stern and Gerlach Experiment with the expected classical and quantum results. Experiments on silver atoms confirm the quantum nature of particles.

oriented at any possible angle with respect to the external magnetic field. When an inhomogeneous field is employed, the small magnet is subjected to a force as shown in Figure II.29. Classically, the result should be a spread of the beam as shown in the figure. The actual experimental result obtained by Stern and Gerlach is described as quantum. The experimental results showed that the magnet associated with the silver atom actually has only two possible orientations with the direction of the magnetic field. When the field is in the horizontal plane for example, the beam still splits into two parts in a direction perpendicular to the field. Thus we have a quantum mechanical particle with a spin direction $\uparrow$ or $\downarrow$. These are called spin up and spin down directions. Thus one has a quantization of the values of the spin. (The magnitude of the spin defines the number of possible orientations).

It is possible to think of the Stern and Gerlah apparatus as similar to the polarizer for photons. It is possible also to arrange the system such that only one of the two possible orientations, either the $\uparrow$ or $\downarrow$ exits from the apparatus. The other is blocked. If two such systems with magnetic fields oriented in opposite directions are used, the beam should not pass

though both. We are assuming that the electrons have a particular direction of spin. The ↑ (spin up) particles passing through the first cannot pass through the second since it permits only ↓ (spin down) particles to pass. This description emphasizes the similarity with the polarizers. As with polarizers, a series of these units can be imagined with the beam passing through all of them. These can be called electron polarizers. Then one notices that the net number of particles passing through three of these polarizers with the outer two being oriented in opposite directions and the center one at an angle è is given by the same cosine law that explains the intensity of light passing through three polarizers. Since beam consists of particles, as with photons, it becomes necessary to assume that the spin of the particle (the quantized value) cannot be defined before the measurement is actually performed.

The most interesting aspect is that while the crossed polarizers, those which will not allow any light to pass through are oriented at 90º to one another, the crossed Stern and Gerlach units are oriented at 180º to one another. This is fundamentally the reason why in quantum theory an electron is a quantum entity that is in an identical state after a rotation of 4ð rather than 2ð for a photon. The above experiments confirm the final attributes of individual quantum particles, namely their classification into fermions and bosons based on the experimentally observed property of spin which has no defined value till it is experimentally measured. The quantization of spin can also be related to the Heisenberg uncertainty relation. Precise knowledge of the spin of an electron would also violate an uncertainty relation. All components of spin cannot be known simultaneously.

The series of experiments outlined here confirm that the basic quantum description is experimentally valid. Photons and electrons are quantum entities that can sometimes behave as waves and sometimes as particles. The experiments can be made more precise and they appear to be neither or both. In addition to this basic complementarity, it is observed that the property of spin of the quantum particle is experimentally

measurable and at the same time must be assumed not to have a fixed value till it is measured.

So far we have described experiments that determine the properties of the constituents of matter and radiation. Most interestingly, the quantum particle description invariably agrees with the classical description in all cases (even when the velocities are large enough for corrections imposed by special theory of relativity to be relevant). For example, reflection, refraction and interference can be explained on the basis of a quantum description as also a classical description of waves. The results of Thomson's experiment and Millikan's oil drop experiment are compatible with a classical description of the electron being a particle, as also with being a quantum entity. However, some other experiments cannot be described assuming the electrons and photons to be classical particles or waves. In other words, there exists another duality. All results which are "explainable" on a classical framework are also explainable on a quantum basis. The reverse is obviously not true. This tendency to get classically meaningful and intuitively obvious results is very relevant for making philosophical arguments in the final chapter.

III
Collective Behaviour of Quantum Particles

Quantum behavior belongs to the realm of the very small such as for an electron which has a mass of ~10^{-30} kg. While it is true that a human eye well adapted to darkness can detect a bunch of 10^{2-3} photons if they are collected in 0.1 sec, the number of photons emitted by a candle in a second is truly astronomical. It is stated that a candle is just visible at a distance of 10 miles or 15 km. Assuming therefore that a candle releases 100 photons per sq mm every second at that distance, the total number would be 4ð$R^2 \times 100 = 10^{17}$ photons. It is common to present the counter-intuitive ideas of quantum mechanics and then contrast them with the (classical) behaviour of large objects of everyday experience.This approach of introducing quantum phenomena hides the extent to which quantum mechanics is the basis for all macroscopic (classical) phenomena. When we consider the feel of a solid object, the combustibility of coal, the contrasting heat-carrying abilities of copper and wood or even the working of the computer, one is experiencing the quantum nature of a collection of quantum objects. The range of phenomena explained by quantum theories is truly astounding. Properties of the macroscopic world can only be explained using the quantum theory. More importantly, most exciting technological developments of the last 80 years are underpinned by the quantum-theoretical understanding developed during 1930-40.

A qualitative description of the collective behaviour of quantum objects is made here as a basis for appreciating the technological developments. The two types of quantum particles are the fermions and the bosons. In order to understand the macroscopic phenomena, we have to consider a number of fermions combining together to form composite

objects. These behave like independent quantum objects that could be either fermions or bosons. Formation of new quantum objects has to be assumed for understanding all the phenomena that are in our experience. In the process, a veritable zoo of other quantum particles emerges. As shall be described in this chapter, these composite quantum entities have been experimentally confirmed to exhibit the entire range of quantum properties.

Fermion and Boson behaviour

It has been mentioned earlier (in Chapter I) that the existence of two classes of quantum objects: fermions with half integral value of spin and bosons with integral spin, is a basic consequence of the application of the principle of special theory of relativity to quantum theory. It was also pointed out that the fundamental difference between the two classes of quantum particles was the symmetry. While a photon behaves like an object that returns to the original state following a rotation of 2ð, an electron behaves as if it returns to the original state following a rotation of 4ð. The macroscopic behaviour, the physics that emerges when large numbers of quantum particles are involved, depends intimately on this fundamental difference between fermions and bosons.

It is interesting to note that the existence of two and only two types of quantum particles emerges from the uncertainty (introduced in Chapter I) inherent to all quantum particles. Consider the simple case of two quantum particles located at locations 1 and 2. After a short time

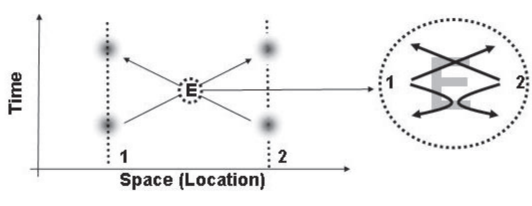

Figure III.1 Schematic view of particle exchange and the uncertainty in confirming exchange at "E".

interval, let us consider the possibility that the two particles are exchanged, that is the particle which was originally at location 1 is now at location 2 and vice versa. The Heisenberg uncertainty relation however means that it is not possible for this statement to be confirmed. Consider Figure III.1. At the crossover point, it is not possible to observe the two particles with indefinite accuracy. Thus the two scenarios shown in the inset (The two possible paths, 1➔2 and 1➔1) cannot be distinguished. This is the reason the term "identical particles" is employed in text books on quantum mechanics (There is nothing to distinguish between the two particles). Incidentally, when we consider a classical system, the particle can in fact be followed with unlimited accuracy and thus it is possible to distinguish the two paths. The classical particles obey Maxwell-Boltzman statistics in contrast to the quantum statistics (Fermi-Dirac or Bose-Einstein). The phenomenon considered in this chapter cannot be explained by assuming the existence of such particles.

At this point, the concept of a wave function has to be introduced. Using quantum theories, only the probability of the result of an experiment can be calculated. The most popular approach is the use of the wave function ø which is calculated from a knowledge of the physics of the system, basically the kinetic and potential energies of the various quantum particles. The wave function ø is a complex quantity that depends on the location in space and time. A complex number is familiar to the students of high school mathematics and consists of a "real" part and an "imaginary" part. It is typically written as (a + ib) where i is the square root of (−1). Since no real number multiplied by itself can be a negative number, i is called imaginary. (a - ib) is called the complex conjugate. The product of a complex number and its complex conjugate is real $(a + ib)\times(a - ib) = a^2 + b^2$. The wave function is the complex quantity defined to ensure that øø* (ø*is the complex conjugate) is probability of occurrence of an event or result of an experiment.

Now consider repeating the exchange described earlier a second time. The situation is identical to the original situation and therefore the physics cannot alter. All the information concerning the particle that can be

experimentally determined is contained in the wave function ø. Therefore, it must be identical following the two exchanges. This is only possible if the wave function obeys one of the two following constraints for single exchange.Either the wave function is unchanged ($\text{ø}_{12} = \text{ø}_{21}$) after one exchange or it changes sign due to exchange of particles ($\text{ø}_{12} = -\text{ø}_{21}$). Bosons are particles whose wave function does not change under exchange and the wave function is said to be symmetric. Fermion wavefunction changes sign under exchange and is anti-symmetric. Two particles with anti symmetric wave functions (fermions) cannot exist at the same location, since one exchange should change the sign, ($\text{ø}_{11} = -\text{ø}_{11}$) and obviously ø =0. The probability of occurrence of two fermions at the same location is zero. On the other hand many bosons can occupy the same point in space. These two properties are respectively the famous Pauli Exclusion Principle and the Principle of Laser Action.

The photon and the electromagnetic force

To understand the behaviour of a collection of fermions, in addition to the Pauli Exclusion Principle, the charge of the particle has to be considered. An electron for example is a fermion with spin ½ that also has a charge 1.69×10^{-19} C. A proton carries an equal magnitude of charge but of the opposite type. Franklin's dictum "like charges repel and unlike charges attract" has been used in areas far away from the science of electric fields. The repulsion between two electrons and the attraction between a proton and an electron are important for considering the properties of a collection of fermions. The force between the objects can be measured. This is called the Coulomb's law which states that the force between charges is proportional to the magnitude of each of the charges and varies inversely as the square of the distance separating them. The force doubles if one of the charges is doubled but decreases to one fourth of the value if the distance is doubled. A little bit of thought results in the famous puzzle about action at a distance. How does one particle "know" about the existence of the second for the force to be generated?. The law gives quantitative value of the force but fails to explain, how the presence of the second charge is detected. The field description was originally introduced

by Faraday for explaining this. This assumes the presence of an electric field around all charges and a magnetic field around magnets, coils carrying current etc. Since the descriptions in this book are quantum mechanical, it is necessary to mention that the attractive and repulsive forces, and also the familiar fields (electric and magnetic), are the consequence of the exchange of photons which are called virtual photons. A descriptive account of the generation of both attractive and repulsive forces by the emission and absorption of photons is provided in Chapter V when experiments confirming the existence of virtual photons are discussed. In this chapter we shall simply continue to use the classical idea of repulsion and attraction between sub-atomic particles. For completeness, it may be mentioned that Einstein provided, in the theory of general relativity, a field description of gravity. However, no "quantum theory of gravity" is currently available and hence gravity is not discussed in this book.

Wave nature of condensed photons

Since a number of bosons can be in the same place, a collection of bosons can exhibit wave nature in two distinct ways. These two have already been mentioned in the last chapter. Experiments performed to verify the wave nature of light used both ordinary lamps and a laser. Observing interference effects using ordinary light is difficult. It is necessary to subdivide light from a single source. In contrast, interference can be easily demonstrated using a laser and can even be observed using two independent laser sources (using special experimental methods). The "coherence time" of the ordinary source of light is much smaller than that for a laser. The easiest way is to understand "coherence time" is to imagine light to be a mixture of waves with random phases. A schematic description is provided in Figure III.2. The distance between two successive sudden

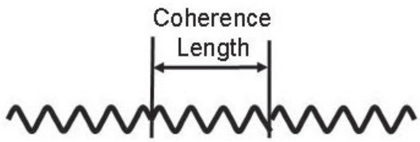

Figure III.2 Simplified definition of coherence length in the wave description of light.

interruptions in the wave is the coherence length and when this is divided by the velocity of light, one obtains the coherence time. After each interruption, the wave resumes with the approximately the same wavelength but with a different phase. This is however an idealization. In reality, the changes are continuously distributed. After a certain length of the wave, there is no fixed phase difference between the two points and coherence is not observed. For performing an interference experiment between two light beams, the “path difference” (difference in distance traveled by the two beams) has to be smaller than the coherence length, obtained by multiplying the coherence time with the velocity of light. While using an ordinary source of light, the experimental dimensions such as the distance between the two slits have to be made very small to ensure that the light from the two sources is coherent. That is, there is no random change in phase between them. (This is a wave description of interference experiments; Feynman’s book provides a quantum description of all these experiments). With a laser source, the constraints are far less stringent since the coherence time (and length) is large. For the same reason, the number of fringes observed with a laser in such experiments is orders of magnitude larger than those observed with ordinary light.

In addition to a large coherent time, laser light has other remarkable properties which are more easily observed. The light from a modern laser pointer does not diverge. In contrast the light from an ordinary flashlight cannot form a sharp beam. The divergence is quite obvious. Using high precision lasers, it is possible to observe a laser beam reflected from the surface of the moon. This method is used to accurately determine the distance between the earth and the moon.

The laser is an acronym that stands for Light Amplification by Stimulated Emission of Radiation. Stimulated emission of bosons is a consequence of the presence of multiple photons in the same state. In the laser, a large number of photons are coherent; they travel in the same direction and are in phase. Stimulated emission is the cause of the many remarkable properties of a laser. Multiple bosons, occupying the same state, are also called “Condensed Bosons”.

To understand the reason for this directionality of the stimulated photons it is necessary to analyze the consequences of reversing the time in stimulated emission. Stimulated emission refers to the excited atom releasing the energy available with it, in the form of a photon, only when another photon stimulates it. In contrast, spontaneous emission does not need a second photon as in the case of ordinary sources of light. This is shown schematically in Figure III.3. Since there is a momentum associated with a photon, (as indicated by the Compton Effect), one has to consider the atom as changing its velocity during emission or absorption. In the picture, the large arrow shows the momentum of the atom and the direction of movement. The excited atom is shown as a star, the de-excited atom as an ordinary circle and the photon as a small circle with an arrow indicating the direction. This arrow also represents the momentum of the photon. The lower part of the picture is time reversed. The direction of motion of the atom and the photon has been reversed.

The first part describes spontaneous emission. The directions of movement of the atom after emission of the photon and the emitted photon are opposite to one another, obeying the principle of conservation of momentum. On time reversal, spontaneous emission becomes spontaneous absorption. If the unexcited atom were initially traveling with a momentum opposite to the direction of the photon, the excited atom will become

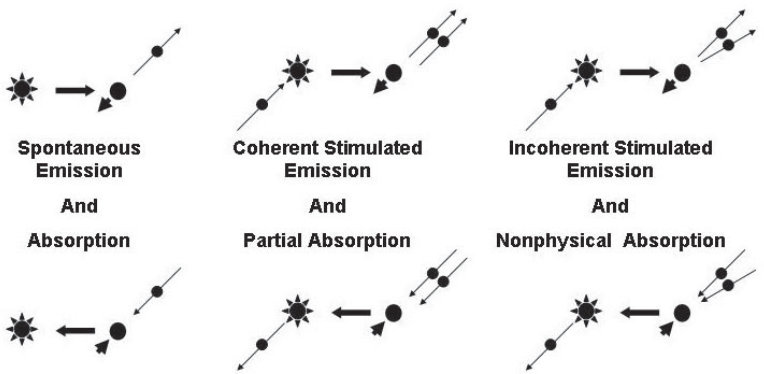

Figure III.3 Schematic view illustrating the coherent nature of stimulated emission.

stationary once again obeying momentum conservation. No physics principles are violated.

Stimulated emission is shown in the second part of the picture. The original photon and the second photon released by stimulated emission are shown to move in the same direction. The momentum of the atom is opposite to that of the emitted photon obeying momentum conservation as with spontaneous emission. When time reversed, two photons and an atom are moving towards one another and one is absorbed. After absorption the excited atom is stationary. Once again there is no obvious mistake with the physics.

If however, the direction of the photon emitted by stimulated emission is not the direction of the original photon, as shown in the third part of Figure III.3, time reversal leads to an anomaly. Obviously, if the atom has to be stationary after the absorption, it should be able to selectively absorb only the photon with appropriate momentum. This selection between photons is obviously meaningless. An atom has no means of selecting the photon on the basis of its momentum. Analyzing stimulated mission from the perspective of time reversal is the only way, of understanding intuitively, the phase coherence of laser light and consequently its directional nature.

When the photon picture is considered, in an ordinary light, the photons come in bunches. In a laser there is little or no bunching, as shown

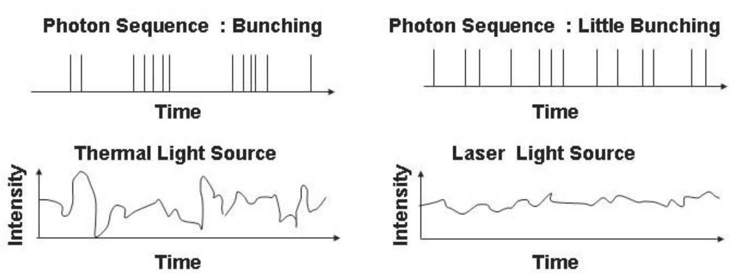

Figure III.4 Photon picture of light sources showing time variation of intensity and photon bunching of thermal light.

in Figure III.4. The intensity of an ordinary light source fluctuates much more than the intensity of the laser. The intensity is large when there is a bunch of photons. The bunching means that the probability for the emission of a second photon after the elapse of a time interval is not large for an ordinary source of light. For a laser, photons are emitted more or less regularly. The probability for the emission of a second photon even after a long interval of time is large. This is the particle description of coherence time.The bunching of photons was also mentioned in describing the experiment of Brumberg and Vavilov in the last chapter. The experiment works only with ordinary sources. With a weak laser, the light would be continuously "observed".

The coherence between photons is also responsible for laser radiation obeying an uncertainty relation between the phase of the wave and the number of photons present. As in all uncertainty relations, the product of uncertainties is larger than the Planck's constant. However increasing the accuracy (lowering the uncertainty) of one of the variables is possible. Recent experiments have selectively lowered the uncertainty in the phase or the number of photons. These are called the squeezed states of light and are expected to be very useful for increasing the accuracy of experiments.

Other bosons and their condensates

As discussed above, a collection of photons behaves like a classical wave in two different ways; the "classical light wave" in which the photons are not coherent and the "laser light wave" which exhibits quantum phenomena at the macroscopic scale. A collection of electrons is not stable since they repel each other. Nature, other than light, consists of matter which in turn is made up of atoms consisting of a positively charged nucleus which contains most of the mass, and a number of electrons attracted to this positive charge. In Chapter II, the classical experiments of Faraday, Avogadro and Dalton were described as a demonstration of the particulate nature of matter, before the smaller particle "the electron" was introduced. This simple picture of an atom as consisting of a nucleus where the positive

charge is confined, interacting with the electrons immediately raises a question about the forces that bind the positively charged protons together in the nucleus in spite of repulsion. This is explained on the basis of the quark theory according to which a proton is made of three quarks, each of which is a fermion with half integral spin. These are bound together by other particles called gluons. The total proton spin is half-integral and it is a fermion. The nucleus, the collection of the protons and neutrons (the second particle in the nucleus which carries no charge but contributes to the mass), behaves like a quantum object. The net spin of a nucleus or an atom can be integral, (when the number of fermions is even) and a boson character is observed. If the total number of fermions is odd, the particle is a fermion and shows the expected behaviour. The silver atom with which the Stern and Gerlach experiment described in the last chapter was performed behaves like a fermion.

A collection of two protons, two electrons and two neutrons is the helium atom which behaves like a boson since there is no net spin. A helium atom therefore exhibits a coherent state just like the photons in a laser beam. There is a difference in that the laser is a high energy state. This will be discussed in chapter IV. A large number of helium atoms go into a single, condensed boson state, of the lowest energy at low temperatures. Below 2.17 K (or about -271° C), liquid helium exhibits a superfluid state with strange quantum properties. The photons in a laser beam show no bunching and there is high degree of order in the photons, as shown in Figure III.4 above. The atoms in liquid helium tend to move in coherence. The result is a liquid that has no viscosity and has an extremely large thermal conductivity. All helium atoms in any sample of liquid helium are not in the condensed state. Therefore, liquid helium, at any temperature other than absolute zero is a mixture of the superfluid, (in which the atoms are in the condensed quantum state) and the remaining liquid in which the atoms are not in the same state.

Heat transport in superfluid helium is fast enough to deliver the heat to the surface which then evaporates. The production of bubbles within the fluid is not possible and boiling stops. Figure III.5 shows the

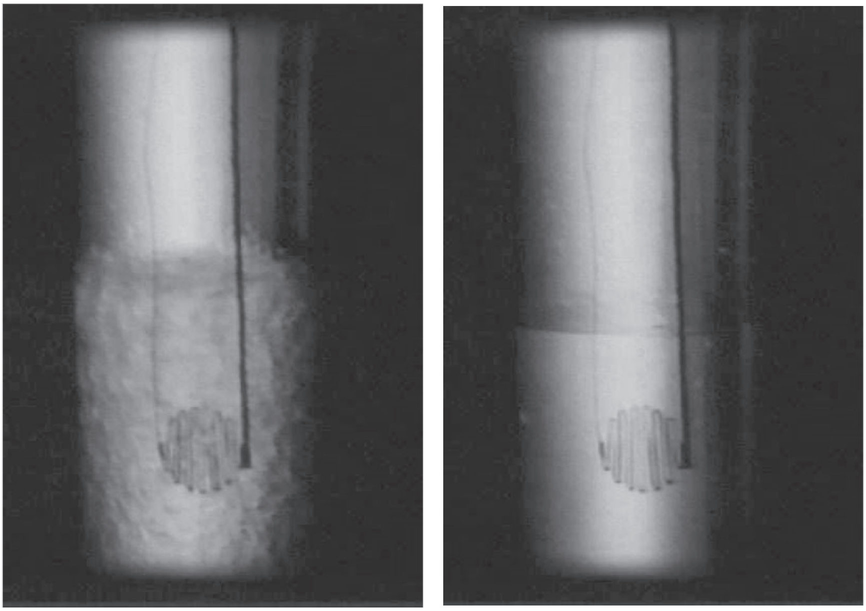

Figure III.5 Lack of boiling in super fluid liquid helium (right) and boiling in normal fluid (left). (Adapted from Sebastien Balibar, "Looking back at superfluid helium", arXiv:cond-mat/0303561 v1 26 Mar 2003. Original pictures of J. Armitage, University of St Andrews)

strange behaviour. At the left, normal fluid is shown. There is vigorous bubbling. The temperature is lowered very slightly and the bubbling completely stops as shown in the right. The liquid evaporates only from the surface, a strange quantum phenomenon like the directivity of the laser beam.

It is common practice to store water in unglazed earthenware pot in spite of the large number of pores. Only a small quantity of water escapes out of these pores since water has a finite viscosity (its evaporation leads to cooling). Superfluid can flow through the smallest pores without any resistance as a consequence of the zero viscosity. Consider two containers, connected by a pipeline which is filled with porous material. When one container is heated, the superfluid will flow from the colder container into the warmer one. Counter flow of the normal fluid back to the first container

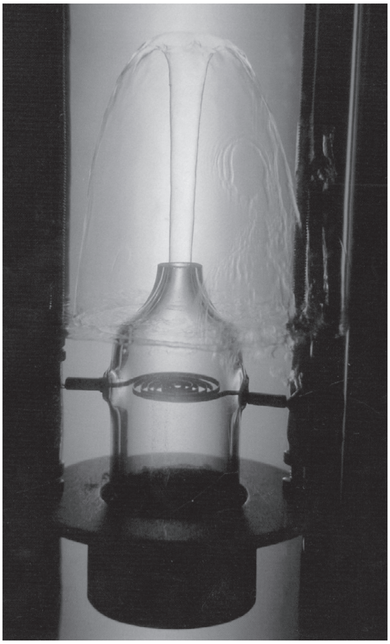

Figure III.6 Superfluid fountain.

is prevented by the porous material. Thus the heater acts as a pump. By proper geometrical design, a fountain shown in Figure III.6 is generated. The liquid is pumped from the inner smaller container into the outer one, through the narrow neck filled with a suitable porous material. Once set in motion, a superfluid will keep moving without any decrease in the velocity. Experiments show that the time required for a rotating superfluid to stop is larger than the known life of the Universe, some 10 billion years. Another peculiar property of this quantum fluid is continuous dripping from filled open containers. In a filled bucket of water, the water level is seen to be higher than the rim by about one millimeter. The superfluid will continuously drip from the bottom of a filled container and the process will continue till the container is empty. The superfluid can flow up the edge in very thin films, because of its lack of viscosity.

In recent years other experiments have spectacularly demonstrated the boson characteristic of atoms. Any atom with an even number of fermions will have a net spin of zero and will be a boson. Many experiments have been performed on ^{87}Rb. This atom consists of 37 electrons, 37 protons and 50 neutrons. A small number of atoms are confined to a small region inside an ultrahigh vacuum container with a density of 10^{9-14}atoms/cc. The temperatures are lowered to ~10^{-7}K, about a hundred billionth of a degree above absolute zero. The Rb atoms then behave exactly as the helium atom in the superfluid state. This is called the Bose Condensate State and is used to confirm Bose-Einstein statistics. It is, however, an even more

beautiful validation of the quantum picture with a collection of fermions constituting an atom behaving like a boson. In addition to condensation, other quantum properties such as interference, entanglement and tunneling have been experimentally confirmed.

The atoms are cooled to the required low temperatures by laser cooling where laser beams of the correct wavelength are made to fall on the atoms from all directions. As mentioned in the last chapter, atoms absorb and emit radiation of a specific wavelength. During the process, momentum is also exchanged between the atom and the photons in the light beam. The wavelength at which an atom absorbs and emits also depends on the velocity of the atom. This is the familiar Doppler effect, which explains why the whistle of a moving train engine changes the pitch as it moves towards and then away from the observer. The wavelength of the laser can be adjusted so that an atom moving towards the laser beam strongly scatters (first absorbs and then emits in different directions) the light. Therefore it experiences a force in the opposite direction to the laser beam. An atom moving in the direction of the laser, weakly scatters the

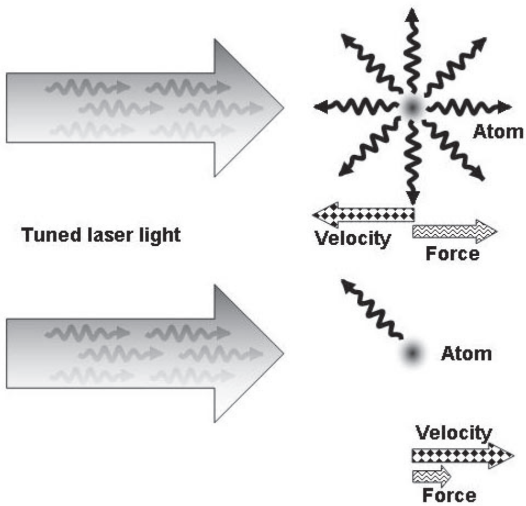

Figure III.7 Principle of laser cooling. The laser wave length is adjusted so that it is absorbed by an atom moving in opposite direction and is slowed by momentum transfer.

laser. (The wavelength is not suitable due to Doppler shift). The net force on these atoms is quite small as shown in Figure III.7. If the laser beams are focused from all six directions (right, left, up, down, forward and backward), the atom experiences a force as it tries to move in any direction making the atomic velocity very small. The lowering of the velocity is equivalent to a lowering of temperature. The difference between a hot gas and cold one is that the velocities of the gas particles are smaller in the cold gas. Usually a magnetic field is also used to confine the atoms at the point of intersection of the six laser beams as shown in Figure III.8. To lower the temperatures further, the magnetic field is designed to enable atoms with relatively large velocities to escape. This process leads to evaporative cooling. Interestingly, evaporative cooling works not only when we sweat on a hot day! A more spectacular demonstration of the condensation phenomenon involving astronomical number of fermions is the state of superconductivity. This will be discussed later in the chapter.

Collection of Fermions

A collection of bosons invariably exhibit a wave nature. This could be the conventional wave nature as represented by ordinary light or may be a quantum wave due to condensation of the bosons as in the case of the laser and a superfluid. This is the consequence of the symmetric wave function of the boson. The particulate nature is revealed when only a few bosons are present. It is already seen that individual fermions can pair up

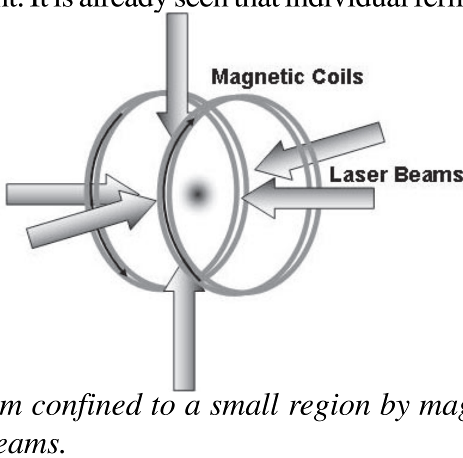

Figure III.8 Atom confined to a small region by magnetic field and multiple laser beams.

into atoms which have integral spin values rather than half integral values which are characteristic of fermions. These atoms then behave fundamentally like bosons. The ability of a pair of fermions, with half integral spins, to be collectively represented by a particle with integral spin is due to the Pauli Exclusion Principle.

Consider the two electrons in a helium atom. The fermions are represented by anti-symmetric wave functions and the two particles cannot be present at the same location. However, due to the attractive force between the protons in the nucleus and the electrons, the electron energy is lowered when it is closer to the nucleus. However, as the electron comes closer to the nucleus, its positional uncertainty decreases. In accordance with Heisenberg principle, the uncertainty in momentum (energy) increases. If two electrons cannot be in the same state, one of the electrons has to be further away from the nucleus. The position of an electron is in balance between the lowering of energy by moving closer to the positively charged nucleus and the enhancement in energy caused by confining the electron to a smaller region of space. In the case of the helium atom, the second electron retains the same energy as the first by reversing its spin. Thus, the two electrons can share the common space which in this case is called the 1s state.

The region in space around the nucleus which can accommodate two electrons with opposite spins is called an orbital. The energy of the electrons in an orbital can take only specific values defined "primarily" by an integer number called the principal quantum number "n". The existence of permitted and forbidden energies is a natural consequence of the quantum approach. One way of calculating the wave function ø in a quantum theory is by summing amplitudes which are oscillatory functions. The square of the total amplitude is the probability. Waves are also mathematically represented by oscillatory functions. Thus, sums of amplitudes can show destructive interference just like waves. In the case of the electrons present around a nucleus, the mathematical calculations show that the probability depends on the energy of the electron and the location. The probability for the presence of an electron of specified energy in a specific region of

space around the nucleus may be very small. These values of the energy are "forbidden". On the other hand, due to constructive interference of the amplitudes, the probability for some regions of space may be large. These regions are the orbitals.

Recent experiments have confirmed the shapes of some of these orbitals. The electron in the atom can be carefully monitored, without causing a significant change in its energy. If this is not ensured, as predicted by Heisenberg Uncertainty Relation, the energy is altered and the electron moves to a different location in space. The electron is carefully and repeatedly detected in the same region of space and the electron density (number of electrons present per unit volume of space) is determined. Experimentally observed electron densities closely resemble the mathematically calculated shapes of the orbitals. For completeness, it should be mentioned that arguments continue as to whether electron density pictures obtained experimentally should be called orbitals. The objection is that an orbital is a mathematical quantity, the statistical probability of detecting the electron with a specific energy in a particular region of space, obtained by a mathematical calculation of øø* and therefore not experimentally detectable.

The principal quantum number determines the number of "nodes" in the orbital. The node is the point, line or surface at which the probability is equal to zero. There can be several different orbitals with the same number of nodes. These differ in the shape or orientation in three dimensional space. The simplest orbital is spherical in shape and is called the "s" orbital. When the principal quantum number is larger than 1, the "s" orbital consists of concentric spheres separated by "nodal" surfaces. As the principal quantum number "n" increases, the electron energy increases. This means that the probability for the electron to be away from the nucleus increases. The other possible configurations are called the "p", the "d" and the "f" orbitals. Figure III.9 shows the shape of the probability in three dimensional space. There are three different "p" orbitals, 5 different "d" orbitals and 7 different "f" orbitals but only one "s" orbital. These differ in the shape or the orientation in space. For example, the p_x

and p_y orbitals are both similar in shape but are oriented at an angle of 90º to one another. The d_{xy} and $d_{x^2-y^2}$ orbitals differ both in shape and their orientation in space. The probability at the point of contact of the lobes is zero and constitutes the node.

The "p", "d" and "f" orbitals have higher energies as compared with the "s" orbital with the same principal quantum number. This is understandable since the electron has a higher probability of being further from the nucleus in these complex "lobes" as compared to the spherically

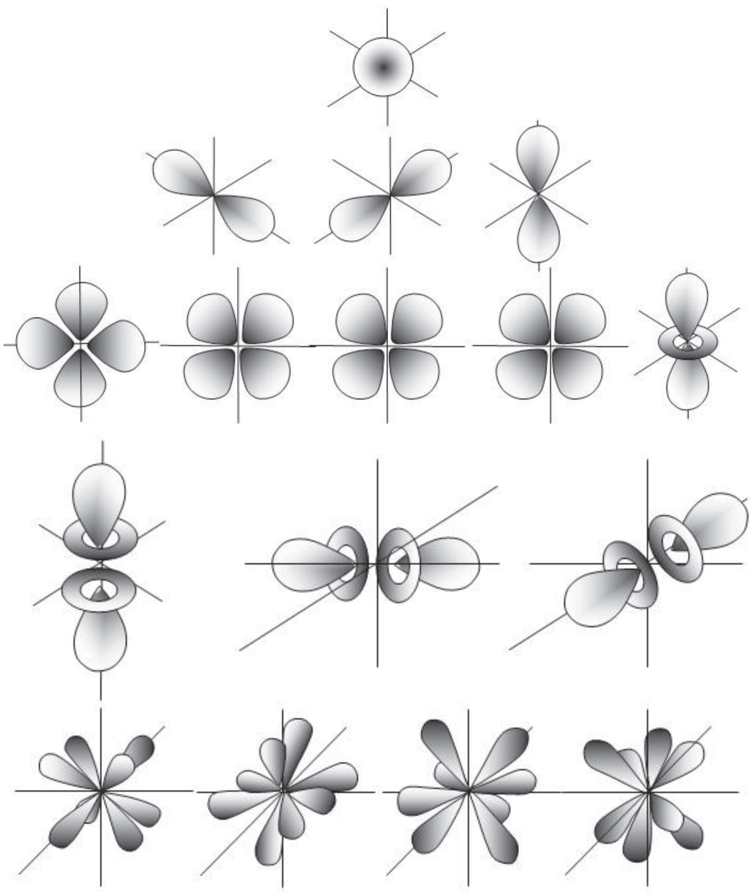

Figure III.9 The shapes of the 1s, 3p, 5d and 7f orbitals.

symmetric “s” orbital. Normally the energies of the three “p” orbitals are all equal. However, in the presence of a magnetic field, the energies differ leading to the magnetic splitting of the energy levels. The levels which are “degenerate” (having the same energy) become “non-degenerate” in the presence of the magnetic field. The reason once again is the non-symmetric shape of these orbitals.

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Building atoms

The energy levels of the various orbitals are shown in Figure III.10. As more electrons are added, orbitals are filled starting with those of the lowest energy namely 1s. The fact that the electron always tries to occupy the lowest energy level available to it means that the 3d orbitals are filled after the 4s levels are filled. In all cases, however, the basic requirement is the same: Lowering of potential energy by coming nearer to the nucleus, while ensuring that the location in space does not have other electrons in an identical state (Pauli’s Exclusion Principle), and that the region of space is not so small that the uncertainty in momentum (energy) would increase due to the decreased uncertainty of location. This orderly filling up of the electron shells explains the periodic table of elements.

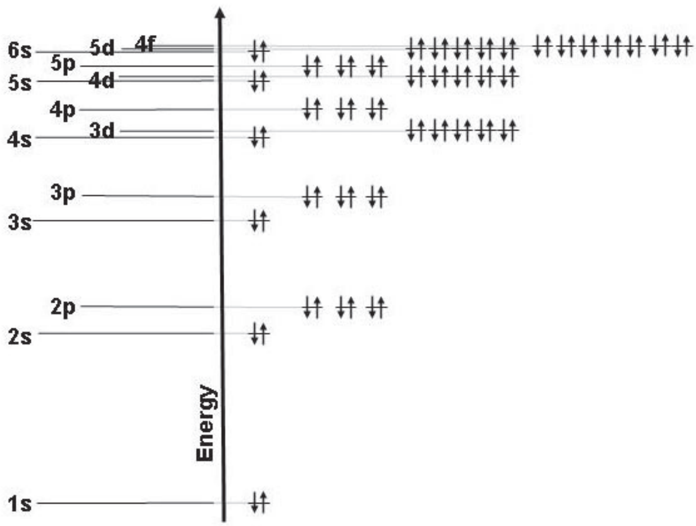


Figure III.10. The energy of the various s, p, d, and f orbitals.

It should be noted that the energy differences between these shells become smaller as the principal quantum number increases. The difference between the 3d and 4s levels for example, is quite small. Also the absolute energy levels of these orbitals are altered by small amounts depending on the number of electrons in the orbitals. For example, the energy level is lowered when there are 10 electrons in the 3d level as compared to 9 electrons. Similarly when there are 5 electrons in the 3d level. These energy changes alter the order of filling up of orbitals. While one expects the 4s levels to be filled before 3d, after $3d_8 4s_2$ (nickel), the order suddenly changes and copper, the next element in the periodic table, has a $3d_{10}4s_1$ configuration, (10 electrons in the 3d orbitals and one in the 4s orbital) rather than $3d_9 4s_2$. Similarly in the case of chromium, a $3d_5 4s_1$ is actually observed rather than $3d_4 4s_2$ after $3d_3 4s_2$ (vanadium).

Different atoms are actually distinguished by their chemical properties. The differences in chemical properties are due to differences in the energy of the outermost electrons. Obviously very small changes in the energy of the electrons located farthest from the nucleus are quite significant. In the example above the properties of copper are defined by the electronic configuration $3d_{10} 4s_1$ which in turn is dependent on minute changes in the energies of the 3d level as it is filled. The properties of copper are significantly different from the properties of nickel, the element with one electron less and zinc, the element with one electron more. Uranium, the heaviest naturally occurring element consists of atoms with 92 electrons, 92 protons and 141 neutrons. As we move from these relatively small number of fermions constituting atoms, towards bulk matter, involving very large number of fermions ($\sim 10^{23}$), small changes in the energy levels become even more important. These small changes are the cause for the bewildering variety of chemical properties and reactions.

Bond formation: Molecules

A filled orbital with two electrons of opposite spin is much less reactive than a single electron. The paired electron can also be "described" as a boson which exhibits only weak interactions with other bosons. The

unpaired electron is a fermion and is more “reactive”. Pairing of oppositely oriented electrons leads to lower reactivity, even as we move, from the individual atom, to chemical bonding between atoms. As we move, from the collection consisting of one nucleus and several electrons to a molecule, consisting of several nuclei and many electrons, one understands the formation of molecules and chemical bonds. The orbitals are called atomic orbitals when electrons from a single atom are considered. When the electrons in a molecule, which is a combination of atoms, are considered these are called molecular orbitals.

In an atom, the potential due to a single nucleus is symmetric and varies inversely as the distance. Thus the energy of the electron is higher when it is farther from the nucleus. Once there is more than one nucleus, the potentials which define the energy of the electron become more complicated. When more than one nucleus is involved, the attraction on every electron due to each nucleus has to be considered. Then there is the mutual repulsion between the nuclei in addition to the repulsions between individual electrons. Uncertainty and Exclusion Principles have to be considered as in the case of individual atoms. The first consequence is the change in both the energy and the orbital shape. When the electrons are localized preferentially on one atom rather than on the other, the situation can be approximately described as transfer of an electron from one atom to another. This is called the formation of an “ionic” bond. Thus, in the presence of a chlorine nucleus in the neighborhood, the 11^{th} electron in a sodium atom will be found to occupy the space around the chlorine atom. We can more conveniently say that the electron is transferred from the sodium atom to the chlorine atom. The electrons in the molecular orbital may exist equally in the region of the two atoms. This is called a “covalent” bond. The two electrons in a hydrogen molecule occupy an orbital. The electron density is equal at both the hydrogen atoms. When a bond between hydrogen and oxygen is considered, the density is not exactly equal. There is more charge near the oxygen atom and the molecule is called “polar”. The permanent excess of electron density at the oxygen can attract the hydrogen from a second molecule of water. This is referred to as the formation of a “hydrogen bond”. Formation of such bonds is crucial for

the molecules of life. For example, the two strands of the DNA molecule are attached into the famous double helix (the twisted ladder) by hydrogen bonds. Thus, the two strands can be separated without spoiling the chain and with very little energy. This enables replication, creating more copies of DNA from the first, a major necessity for life.

Altering the orbitals

Other subtle changes in the shapes of atomic orbitals and their energies are also observed. One possibility is mixing of the basic atomic orbitals. Since energy states are being altered by mixing, this is called hybridization. The best example for complex changes in the energies and shapes of the orbitals is carbon. Figure III.11 shows the various electron energy states in carbon. A pure carbon atom has two electrons in the 1s state and one electron each in the 2s, $2p_x$, $2p_y$ and $2p_z$ states. When the carbon atom is bound to four other atoms, for example four hydrogen atoms in methane, or four other carbon atoms in diamond, the energy states of the four electrons are slightly different. One does not expect the bonds to be identical but observations confirm the formation of four identical bonds. Hybridization of these four electron states results in four new states, all with equal energy. There are once again two electrons in the 1s state and four unpaired single electrons in the so called the sp^3 hybridized states as shown in Figure III.11(a). Similarly, in the case of bonding in a molecule of ethylene, graphite or other carbon compounds having a C=C, there are

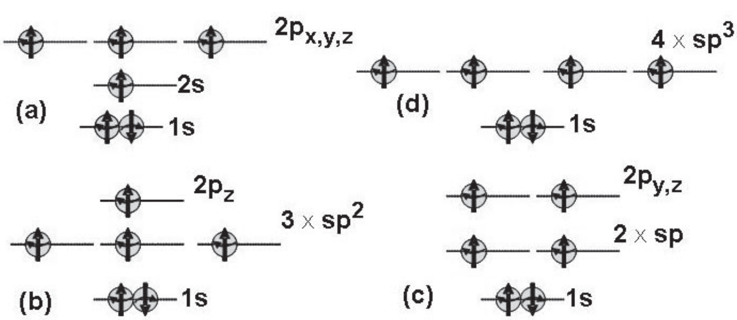
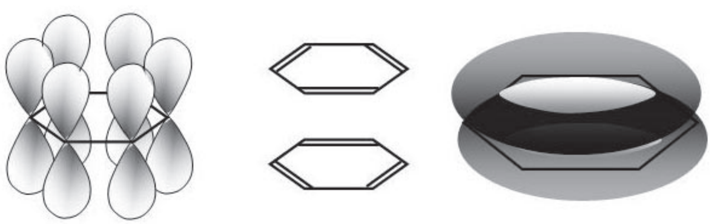


Figure III.11 The four different atomic orbital configurations of carbon atom that are observed in various chemical compounds.

bonds, different from the carbon-carbon bond of diamond that are observed. Here, one electron is left in a p_z state in each of the carbon atoms while the three other electrons occupy the sp^2 hybridized states (Figure III.11(b)). In the case of a molecule of acetylene or other molecules having C≡C bonds, two electrons are left in the p_y, p_z states while two are in the sp states. The sp^3 hybridization in diamond fixes the direction of the bonds as 109° to each other. The sp^2 hybridization of graphite provides three bonds which are at 120° to one another in a single plane. The bonding with the remaining p_z electron is of very low energy. Obviously the presence of other atoms has resulted in large scale alteration of the atomic orbitals of carbon.

An even more subtle change called resonance is also observed. This is best illustrated by studying the properties of the benzene molecule. As mentioned above, the carbon atoms in a benzene molecule have the p_z orbital free while the other three outer orbitals are hybridized into the three sp^2 hybrid orbitals. These are in a plane with an angle of 120º between them. These form the sigma bond framework which is a regular hexagon as shown in Figure III.12(a). The overlap of the p_z orbitals results in the formation of the ð bond between neighboring carbon atoms. In a benzene ring, this can occur with either the neighbour on the left or the right leading to two possibilities which are mirror images of one another as shown in Figure III.12(b). Experiments, however, show that the carbons atoms in the ring do not have different types of neighbors (with a single bond on one side and a double bond on the other). The ð electron cloud forms a



F*igure III.12 Structures of benzene. (a) The ring of sigma bonds and the pz orbitals. (b)The two possible bond structures and (c) The observed resonance hybrid.*

closed loop above and below the planar ring as shown in Figure III.12(c). This is said to be the resonant structure of the two possibilities and preferred because of a lower energy. The reason for this lowering of the electron energy is very important. As the electron moves away from the nucleus of an atom, its energy increases and thus it would prefer to be in a region closer to the nucleus. In an atom, this would lead to smaller value for the positional uncertainty and increase in the uncertainty in energy. These tendencies are balanced. When a molecule is considered, it is not necessarily true that the potential energy of the electron should increase since it may get closer to a second nucleus. Thus a larger orbital size would lead to a lowered and preferred energy.

Free electrons in condensed matter

The resonance argument is extremely useful in understanding the behaviour of extremely large number of fermions, comparable to the Avogadro Number ($\sim 10^{23}$). If we consider the gaseous state, quantum phenomena are only observed at the molecular or atomic level and there is simply a collection without any novel properties. The inter-atomic distance in a gas at STP is ~3.3 nm which for a molecule of ~0.1 nm size is extremely large and there can be no influence of the electrical attraction of the second nucleus. On the other hand in condensed matter, which includes both solids and liquids, the inter-atomic separation is almost equal to the atomic size and the atoms are in intimate contact.

Figure III.13 shows the potentials and the electron energy levels of a collection of atoms. The atoms are shown to be equidistant to one another but this can be ignored. Extending the extremely simple argument

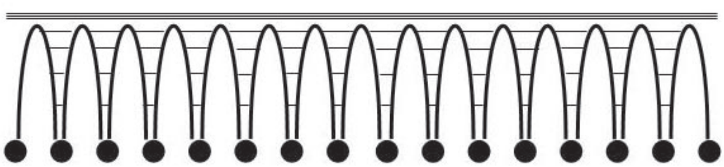


Figure III.13 The electric potential near the atomic nucleus shown in thick lines and the electron energy levels of a row of atoms.

used to explain resonance, it is possible to argue that the electron energy levels at the top of the potential wells can be completely delocalized. These have been shown as continuous energy levels for the entire row. The levels near the bottom of the potential wells are shown localized to the specific potential wells and hence are localized on one specific atom. The size of the electron orbital, for the delocalized electron levels, is as large as the row of atoms. However, the Pauli Principle is still operative. The electrons are fermions and only two electrons can occupy one state even if the orbital is very large. In both the small 1s orbital which is about 0.1 nm in diameter and the delocalized orbital in a piece of silicon, (which can be several cm in size), where the orbital is as large as the piece of silicon, only two electrons with opposing spins can be accommodated. This shows the importance of Pauli Principle in comparison to inter-electron repulsion! As the number of atom increases, the number of electron states correspondingly increases. When there are $\sim 10^{23}$ atoms in a sample, there are a very large number of electron states and these become closely spaced bands.

Thus we can conceive of two types of electron states in a condensed material, where the atoms are close to one another, localized and delocalized (extended). Electrons in the localized states are associated with one or at most a small number of atoms in the vicinity. The electrons in the delocalized states are free to move in the entire (macroscopic) sample of the material. As was pointed out by Mott in his Nobel lecture, transparency of ordinary glass and the ease with which it can be colored by introducing a small quantity of impurity shows the absence of localized electron states in glass. If there were large number of localized states, the photons of corresponding energies would be absorbed. This is similar to the earlier observation of the atoms absorbing radiation in sunlight, leading to spectral lines. Glass in contrast is quite transparent down to about 300 nm which suggests that there are no states which can absorb photons of smaller energy.

The electron in a delocalized state is free to move within a macroscopic body such as a crystal of silicon. However, the actual

movement of such an electron under the influence of an electric field cannot be observed in silicon but can be observed in a metal. The energy required to move an electron is small if there are vacant electrons states available nearby into which the electron can be excited with a small amount of energy. It should be noted that at absolute zero or the lowest temperature, all electrons occupy the lowest available energy levels. At any finite temperature, for example room temperature, the electron has energy (corresponding to room temperature, called thermal energy) and can move into a higher energy level provided these are available.

It can be observed that mercury is a liquid metal that easily conducts electricity and therefore has delocalized electron states. It is important to note that both mercury and glass are materials where the atoms are not organized into any specific order. It is intriguing that the existence of the large number of atoms results in an average influence that makes the electron free to move anywhere within the macroscopic material.While the delocalized electron states can be qualitatively inferred from experimental results for disordered materials, it is necessary to consider perfect order in order to mathematically derive these states. The mathematical description is called the Bloch State and it can be shown that in a perfectly ordered solid, the delocalized states form bands of delocalized states. The lowest band of such states is called the valance band and the next higher band is called the conduction band.

The state of these bands defines the electrical conduction of materials. The distinction between metals such as copper and insulators

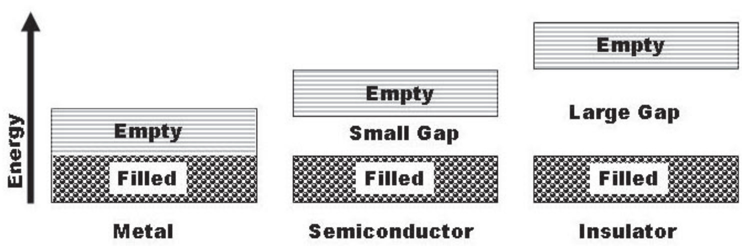


Figure III.14 The valence and conduction bands in metals, semiconductors and insulators.

like diamond is the energy required to move an electron from one delocalized energy state to another. In a metal, at absolute zero, the electrons partially occupy the conduction band. Some of the electron states in the conduction band are unoccupied as shown in Figure III.14(a). Therefore empty states are available for an electron even when the energy (the applied voltage) available is very small. In an insulator, the filled levels are separated from the empty levels by a large forbidden gap. The valance band is completely filled while the next band, the conduction band is completely empty at absolute zero of temperature. The energy required to shift an electron to an empty level is so large that only high energy photons can provide the required energy and thus it is not possible to drive a current by applying an electric voltage. A semiconductor, as the name itself suggests, lies somewhere in the middle. The forbidden gap is small enough so that energy available at room temperature is sufficient to excite a few but not many electrons into empty states in the conduction band. They can then conduct electricity since empty states are easily available.

In a semiconductor, the number of carriers are about a thousand to a million times less in number than in a metal. This number can be altered in a controlled way by introducing impurities in the material. These impurities create electron levels in the energy gap. These are localized states. In an amorphous semiconductor, for example selenium, there are large number of such localized states, introduced by disorder in the material rather than impurities. States formed close (at an energy difference much smaller than the band gap) to the top of the valence band (normally filled) or the bottom of the conduction band (normally empty) are very useful. Under these conditions, the energy required to excite electrons, from these defect states to the conduction band or from the valence band to the defect states is low and almost all available electrons are excited.

The impurities are chosen so that the defects close to the conduction band have an extra electron for example by substituting a phosphorus atom with five electrons in the outer orbitals, in place of silicon which has four. The number of electrons added to the conduction band is equal to the number of phosphorous atoms introduced which is under experimental

control. The concentration of electrons in the conduction band is enhanced and the material is called an "n" type semiconductor since it has a large number of negatively charged carriers of electricity. Similarly, an element with three outer electrons such as boron replaces silicon to introduce a defect state near the valence band. Now the electrons from the valence band are excited into these defects, leaving vacancies or holes in the valence band. Due to this excitation, the valence band is no longer completely filled. There are electron vacancies or "holes" into which electrons form the valence band can be excited with very little energy and contribute to conduction. This conduction is due to the positively charged vacancies or holes and the material is called a "p" type semiconductor.

When the n and p type semiconductors are placed in contact, the electrons from the "n" type semiconductor diffuse into the "p" side since more electrons are available. Similarly, "holes" diffuse from the "p" to the "n" side as shown in Figure III.15. These "diffused" electrons and holes neutralize (the electron fills the vacancy) each other at the interface region which therefore has no free charges but a small electric field. This is called the depletion width. The p and n type materials are normally neutral; they have equal number of positive and negative charges. When the n type material loses some electrons, as described above, it develops a net positive charge which opposes the further motion of the holes from the p type semiconductor. Similarly the p type material develops a negative charge. These charges cause the electric field in the depletion region. This extra field is represented as a bending of the bands in Figure III.15. The electrons

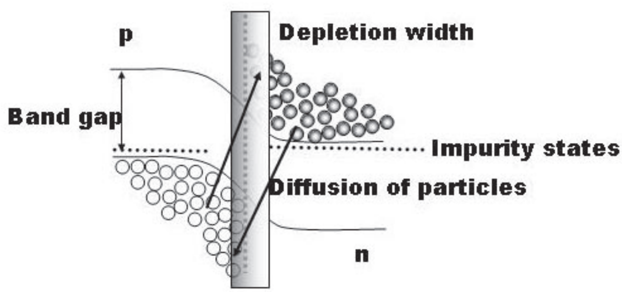


Figure III.15 The formation of a rectifying p-n junction.

in the n type material would then require extra energy to move into the higher conduction band of the p type material. An external voltage in the appropriate direction (positive terminal of the battery to the p type) attracts the electrons over the barrier leading to conduction. An opposite voltage causes no conduction. This is the familiar semiconducting rectifier or diode.

The amazing thing is that this description of a device actually fits experimental observations, not withstanding all the assumptions and simplifications. The bands formed due to the quantum mechanical description of the electron states are treated as potentials. Electrons and holes are described not merely as particles but as "classical particles" without any quantum mechanical uncertainty. This most amazing mixture of classical and quantum descriptions is very robust. The entire semiconductor device development normally needs no other quantum ideas. The brief description of this semiconductor physics has been included here to expose this hybrid theoretical approach.

Magnetic properties of free electrons

Many properties of condensed matter can be understood by treating the material as a collection of tiny classical magnets after introducing the quantization of spin and orbital properties. This closely corresponds to the description of a semiconductor device in terms of the properties of a collection of free electrons in a potential defined by quantum mechanics as discussed above. Since the electron is the carrier of charge, even if the electron is considered as a classical spherical object with charge, both its movement around the positive charge of the nucleus and its spinning result in magnetic moments. The magnetic moment of the electron in motion can be calculated by assuming the electron movement as a circle, resulting in a current and a net magnetic moment. The value of the magnetic moment calculated in this classical picture is experimentally confirmed. However, a classical magnet can be oriented at all directions and the energy varies continuously. But actually, only quantized orientations are observed. Earlier, it has been mentioned that the three "p" orbitals have three different energies in the presence of a magnetic field. Quantization of the spin magnetic

moment was introduced when the Stern and Gerlach experiment was described. Thus in describing the magnetic properties of a collection of fermions in condensed matter, the quantization of the orbital magnetic moment has to be included.

Spin was introduced as a characteristic property of all quantum objects that determines its classification as a fermion or a boson. The total spin of an atom is the vector sum of all the spins associated with the nucleus and the electrons. Unlike orbital magnetic moment, the spin magnetic moment is experimentally half the value that would be expected if the electron was considered as a rotating charged sphere. This correct value can only be calculated using Dirac equation, bringing the spirit of relativity into quantum theory. The anomalous factor of 2 is called the Lande "g" factor and is one of the most important numbers in quantum theory. Its experimental measurement confirms that the electron is indeed a particle with a rotational symmetry of 4ð. The experimental value for the Lande "g" factor of 2.0023193043737 reflects the corrections introduced by the emission and absorption of virtual photons which will be discussed later. It can both be calculated and experimentally confirmed to an accuracy better than 1 part in 10^{11} demonstrating the strength of quantum theories. The net quantized magnetic moment of the atom is obtained by vector sum of the spin and orbital contributions.

The first magnetic property that is easily understood by assuming a collection of free quantized spins is paramagnetism. Due to thermal energy, the spins are disordered and are oriented in random directions. The energy scale in this case is ~10^{-4} eV. An external magnetic field would align these magnets but once the external field is removed, the thermal energy available at room temperature is sufficient to induce disorder. If this property is investigated at various temperatures, the induced disorder is quite clearly dependent on the temperature. It is possible to calculate the magnitude of the alignment of spins due to a specific magnetic field at a given temperature and compare them with experimental results and confirm that the condensed material behaves like a collection of tiny magnets. This is true, in spite of

the experiments already cited, that the spin of a quantum object has no real value unless it is measured.

Similarly, Pauli Paramagnetism can be explained as the property of the collection of magnetic moments of the free electrons in metals. The spins of the individual electrons (rather than of the atoms as in the case of paramagnetism) are aligned in the presence of a magnetic field. Each of the energy states has two electrons with opposite spins. Figure III.16 shows the conduction band split into the “spin up” and “spin down” parts. The reason for the parabolic shape will be discussed in the next subsection when “nearly free electrons” are introduced. The energy of the spins parallel to the external field is lowered while those of the spins aligned opposite to the external field are raised as shown in Figure III.16. Thus, there are vacant “parallel” spin states with lower energy into which there is a transfer of electrons with “anti-parallel” spins, as shown in the figure. Thus the lowest energy state, in the presence of a magnetic field is one where there are more electrons parallel to the external field. The energy per spin is once again ~10^{-4} eV. Theory, considering electrons as free spins, and experiments can be compared quantitatively. However, unlike the case of paramagnetism of atoms, the effect is very weak in metals and is also temperature independent. This is again a consequence of the Pauli Exclusion Principle. As shown in picture, only the top few electron states (of width corresponding to the thermal energy) participate. The rest of the electrons are not affected by the magnetic field. Consequently, Pauli paramagnetism is very weak and since the number of electrons transferred changes very little with temperature, is independent of temperature.

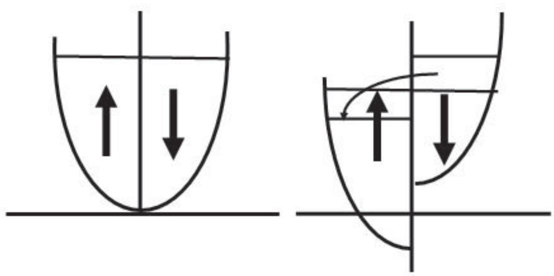


Figure III.16 Influence of magnetic field on the energy bands of free electrons.

Another magnetic phenomenon that can be easily understood on the basis of free spins is diamagnetism or the tendency of materials to oppose the application of magnetic field. Classically, this can be modeled as the motion of individual charges, creating a magnetic field in a direction opposite to that of the external magnetic field. When the magnetic field is caused by electrons confined to individual atoms, this is called Larmor diamagnetism and when caused by the movement of free electrons in a metal, it is called Landau diamagnetism. The magnitude however is much smaller ($\sim 10^{-9}$ eV) than paramagnetism. Quantitative comparison between experiment and theory can be achieved by simple modeling of free spins. This weak phenomenon is the source of another mystery called the "Fractional Quantum Hall Effect" which will be discussed later.

Outside a physics textbook, magnetism is the phenomenon associated with compass needles and electromagnets. It is natural to expect that one tiny magnet would induce the second to align parallel to it leading eventually to a magnet that retains the order among spins even when external field is removed. In this view, "ferromagnetism" as this phenomenon is called in physics, is a simple extension of paramagnetism. In fact, a classical theory was developed on this basis. However, this dipole-dipole interaction varies as the cube of the distance and can be shown to be too small to account for the observed magnetism of materials.

Ferromagnetism is the first phenomenon encountered in the present chapter that cannot be explained by the idea of simple free electrons and spins. There are many other phenomena, including conduction of heat, which cannot be similarly explained on the basis of free individual particles. It is necessary to discuss the limits beyond which these simple ideas are not useful and the reason for this limitation.

Nearly Free Electrons

The parabolic energy bands shown in Figure III.16 are to be expected for free electrons. The positive charge is assumed to be uniformly distributed, ensuring merely that the electrons do not leave a material. For

any given energy, there is only one electron state with two electrons with a wave vector (The number of waves per unit length or inverse of wavelength) defined by de Broglie's equation. The energy band in Figure III.16 plots the energy as a function of the wave vector. In three dimensions, such a plot would resemble a perfect sphere with the states inside the sphere being filled. As mentioned above, for a mathematical derivation of the energy bands, a perfect arrangement of atoms in a periodic lattice is to be assumed. The simplest case is to imagine a single line of atoms, with a constant inter atomic distance, very much like the schematic in Figure III.13. The conduction and valence bands obtained in that case are superimposed on the free electron band diagram in Figure III.17. The single band is split up into many bands the uppermost being the valence and conduction bands as shown.

The electron energy states can be visualized as the interference of electron waves being diffracted by the lattice of positive charges. Some

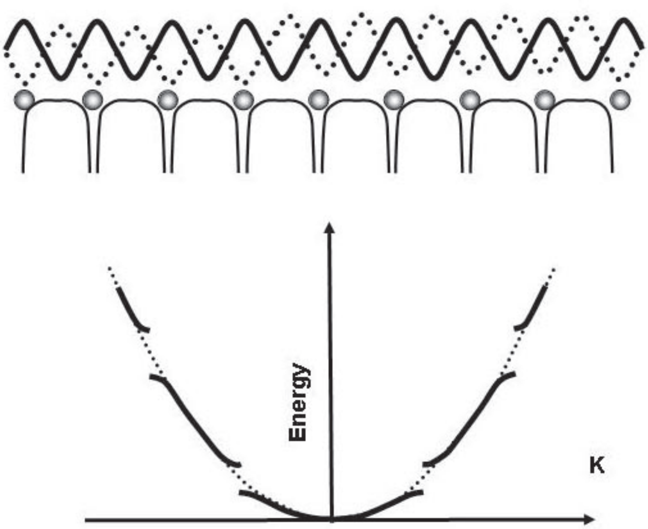


Figure III.17 The energy bands in the presence of a periodic potential. The electron wave with maximum near the atom(full line) has a lower energy than that with the maximum at the mid point (dashed line).

energy levels are forbidden because the electron waves with these energy levels suffer destructive interference. Consider the wave vector of the electrons at the top of the valence band and the bottom of the conduction band. Two electron states with identical wavelengths (wave vectors) still have significantly different energies. The energy of the electron in a periodic lattice is not defined by the wavelength through the de Broglie relation as in the case of a free electron. The key issue is the location of the maximum probability for the electron wave. If the regions of maximum probability are nearer to the positively charged nucleus as in the case of the valence band electrons, the electron energy is lower. Whenever the probability is high away from the nucleus as for the conduction band electron, the energy is high. This is schematically shown for the one dimensional lattice in Figure III. 17.

The situation becomes more complex in the case of three dimensional lattices. The distance between successive atoms in a lattice depends on the direction. Even in a simple square lattice, the distance between points on the diagonal is $\sqrt{2}$ times the distance along the lines. The formation of constructive and destructive interference patterns, leading to the band gaps, depends on the direction. Just as electrons with identical wave vectors can have different energies, it is possible for electrons with identical energy to be represented by states with different wave vectors in different directions.

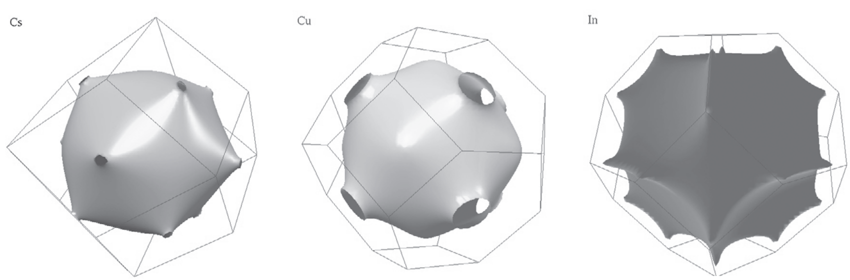


Figure III.18 The Fermi surfaces for cesium, copper and indium. (Used from http://www.phys.ufl.edu/fermisurface/; A Database of Fermi Surfaces in Virtual Reality Modeling Language by T. S. Choy, J. Naset, S. Hershfield, C. J. Stanton and J. Chen, University of Florida)

At absolute zero, the electrons are all in their lowest possible states. These would constitute a sea of electrons with well defined wave vectors. A three dimensional representation of the filled electron states, in different directions called a Fermi Surface can be constructed. As mentioned above, the Fermi Surface for a sea of free electrons is a sphere. Even for the simplest three dimensional crystals, the shapes are more complex. The calculated pictures for three simple metals are shown in Figure III.18. While for the simple metals like cesium and copper, the resemblance to a sphere is quite obvious, for indium with unfilled "d" electron orbitals the picture is more complicated. The shapes have been confirmed experimentally by many methods, for example by measuring the resistance of the surface, in the presence of a magnetic field at very high frequencies (cyclotron resonance) or measuring the energy of electrons released by the material in different directions due to X-rays (angle-resolved photoelectron spectroscopy). Such measurements reveal that the electron energy state with the lowest energy in the conduction band and that with the highest energy in the valance band can occur in different directions in the crystal. This is called an indirect band. The results for diamond, which has the same structure as silicon, is shown in Figure III.19.

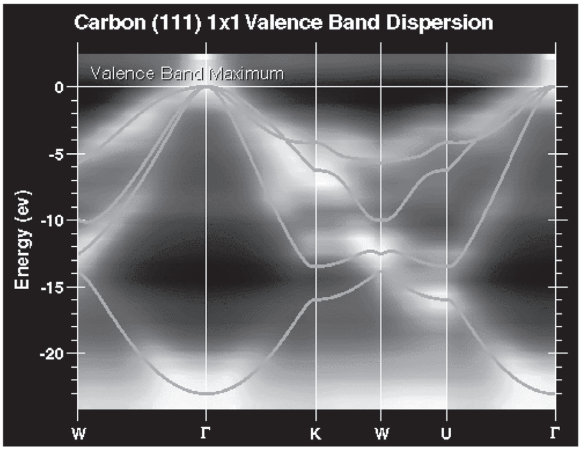


Figure III.19 The band structure of diamond showing the indirect band gap and its confirmation by angle-resolved XPS. (From the data of Dr M. Hochstrasser and J. Tobin Lawrence Livermore National Lab 2002).

The practical consequences of the indirect band gap are enormous. Silicon is the work horse of the electronic industry and all modern computer chips are made using silicon, but a simple light emitting diode cannot be made to replace the energy inefficient incandescent light bulb. Even though there is interest in protecting the environment, suitable silicon solar cells are not available. Silicon is an indirect band gap material. The wave vector of the electron at the bottom of the conduction band is significantly different from that of the electron at the maximum of the valence band. Consequently, when a photon is absorbed in the solar cell, and the electron is excited into the conduction band, some energy is lost as heat (through another quantum object called the phonon to be introduced later). Similarly, the electron in the conduction band of silicon cannot lose its energy to a photon, while returning to the valence band because the difference in wave vectors corresponds to a difference in momentum. Momentum has to be conserved by involving a phonon. Both these problems can be avoided in GaAs, a direct bandgap semiconductor where there is no difference in the wave vectors. However for other technological reasons, it has not been possible to develop low cost GaAs devices.

The properties of semiconductors indicate the central role of concepts such as electron wave interference in understanding physical properties of materials. At the same time, experimentally, it is possible to identify subtle changes in properties caused by the electrons being nearly, rather than absolutely, free. In addition to the complex shape of the Fermi surface, the "effective mass" of the electrons is another interesting deviation from the ideal free electron. Experimentally it is possible to confirm that the electron in a semiconductor appears to have a mass significantly different from that that of a free electron. The mass depends on the curvature of the Fermi surface: sharper the curvature, smaller the mass. In actual experimental work, the complex band shapes are approximated by a parabola near the maximum or minimum of the band. Then the effective mass is mathematically defined. For example the "free" electrons in GaAs have an effective mass ~6.7% of the free electron mass. This low mass is responsible for the use of GaAs transistors in high frequency applications for example in mobile phones. At the other end, heavy fermion systems

with effective mass values of 10^{2-3} are experimentally confirmed. Significant however is the fact that it is possible to collect all the complexities of the real system in terms of a single parameter like the "effective mass" and continue to treat the system of electrons as nearly free.

Broken symmetry

The reason for observing free electron and spin behaviour in condensed matter was first explained by Landau as the consequence of a broken symmetry. Descriptively, his idea points to lack of isotropy as the key difference between an individual atom and condensed matter. An atom is spherically symmetric; it is identical in all directions. Similarly, an assembly of gas molecules is isotropic. A continuum used to describe matter in classical theory is also completely isotropic. In contrast, an atomic picture of condensed matter is not isotropic. Consider a close packed square lattice where each atom is represented by a sphere of diameter "a". In one direction the atoms are repeated after a distance "a". Along the face diagonal, the atoms are repeated after $\sqrt{2}a$. Along the body diagonal, they are repeated after $\sqrt{3}a$. This simple geometry shows that the most important characteristic of a crystal is lack of symmetry. Even in a liquid, experimental studies indicate that the neighboring atoms are ordered regularly. A glass or amorphous solid has similar short range order. This is best illustrated by the model of fused quartz. Each atom of silicon is surrounded on the average by four atoms of oxygen which are approximately at the vertices of a tetrahedron with the silicon atom at the center. Thus as viewed from the silicon atom the immediate neighborhood is not isotropic. In some directions oxygen atoms are present as the nearest neighbors; along other directions they are not.

This lack of isotropy is referred to as "broken symmetry". Landau showed that in the system obtained by broken symmetry, the distribution of energy among the entire set of particles, (the many particle excitations) can be approximately described as "quasi-particles". The properties of materials, which have undergone a transition to the state with broken symmetry, depend on all the particles and the way in which they interact.

Obviously it is the interaction between all the atoms which is responsible for the order displayed by them collectively. Landau's mathematical theory showed that it is possible to mathematically describe the energy states in which all these particles will exist as energy states of a mathematically defined quantum wave. At least the low energy states of the collection of bodies can be represented as the excited states of a "new" quantum particle. These "new" quantum objects can be experimentally investigated. The "quasi- particles" can be both fermions and bosons. They obey the Planck's law if they have no mass and the de Broglie formula otherwise.

In the above example, translational symmetry was broken along the three directions in space. In accordance with the broken symmetry theorem, the vibrations of the lattice can be described in terms of particles called phonons. The energy states of the lattice can then be described in terms of the energy states of the phonons. Above a critical temperature a ferromagnetic piece of iron oxide is paramagnetic. The magnetic moments have no preferred direction. In the ferromagnetic state there is a specific direction along which all moments are aligned. The variation in the alignment of all the moments is therefore described as a particle called magnon. The magnons describe the energy of the collection of magnetic moments. In the case of simple metals, for electronic properties, the quasi-particles are identical to electrons in all properties except for the mass.

The idea of broken symmetry is very powerful. This theorem can be seen as a corollary of the famous Noether's theorem which states that whenever there is symmetry, an associated quantity is conserved. For example in the context of mechanics, motion of the object has translational invariance. It should not matter where the experiment is performed. The physics of the system cannot change. It can be mathematically shown that the law of conservation of momentum is a consequence of translational symmetry. Whenever there is force, the translational symmetry is broken. The physics of the system then depends on the position of the particles with respect to the force. The existence of a broken symmetry is one way of describing the existence of a force. In particle physics, particles such as electrons and force carriers such as photons emerge as the consequence

of broken symmetry. As is well known, there are four forces of nature (i) The electromagnetic (Coulomb) force which is caused by the exchange of virtual photons between charged fermions, (ii) The weak force responsible for radioactive decay, (iii) The strong or nuclear force that confines the protons and neutrons (actually the quarks) inside the nucleus and (iv) Gravitation. The first three are all described by quantum theories. The unified theories treat all these forces as identical at very high energies (temperatures) and being separated at lower energies by broken symmetry. Several examples of quasi-particles relevant for our description of collective behaviour of quantum particles are discussed below. The depth to which the quasi-particles exhibit the quantum nature will be discussed in some depth.

Quantized lattice vibrations

As mentioned above, the Landau theory of broken symmetry can be used to predict that broken symmetry results in the formation of quasi-particles with boson character, called phonons, which are also waves in a crystal lattice. It is necessary to postulate the quantization of lattice

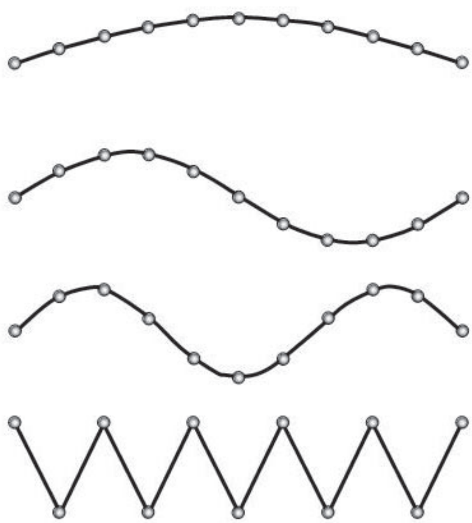


Figure III.20 Schematic demonstrating that the minimum wavelength of a lattice vibration is twice the inter-atomic spacing.

vibrations in order to understand the thermal properties of materials. Historically, Einstein had introduced an ad hoc assumption that the vibrations of solids are quantized obeying Planck's law in analogy with radiation. This assumption correctly described the lowering of specific heats of solids at low temperatures. These experimental results cannot be explained if the lattice vibrations are considered as classical waves. This is similar to the ad hoc quantization introduced by Planck to account for black body radiation.

Einstein assumed that lattice vibrations consisted of a single mode of vibration with a fixed wavelength. Debye pointed out that the minimum wavelength at which a lattice can vibrate is obviously twice the atomic spacing as shown in Figure III.20. The wave moving the lattice can have a wavelength longer than the atomic spacing. The smallest wave will move the nearest atoms in opposite directions. Thus the phonons can have a distribution of possible modes with a maximum limit on the frequency corresponding to the shortest wavelength. It can be assumed that these waves are all quantum waves obeying Planck's law. The model can then be used to calculate the specific heat which is predicted to vary with the cube of the temperature. The Debye model fits the low temperature specific heats even better than the simple Einstein model. The concept of a phonon is essential to understand the difference between thermal conductors and thermal insulators particularly for achieving a quantitative fit with observations.

The phonon cannot be simply assumed to be a theoretical construction. It is a quantum particle and experiments for direct observation of the quantized nature of vibrations in condensed materials and even in molecules can be easily performed. For example, the Raman Effect is a good indication of the quantized nature of vibrational energy of molecules. In the Raman Effect, a photon that is incident on a molecule either absorbs or emits a phonon. In the first case the photon gains energy and in the later it loses energy. This gain or loss is quantized. Only specific quantized values are possible. In case of condensed matter, since the number of states is large, there is a band of energies just like the energies of free

electrons. However, phonon being a boson, there is no restriction on the number of particles in a given energy state. Models such as those of Born and von Karmann are more accurate than the simple model of Debye. These theoretically calculate the phonon spectrum based on the atomic mass and order in which the atoms are arranged in the three dimensional lattice (just like the Fermi surface for electrons). These calculations can be experimentally verified using neutron scattering, high energy X-ray scattering or optical spectroscopy studies. These give a direct confirmation of the existence of phonons as free quantum bosons present in condensed matter. They exchange momentum and energy with the neutrons, X-ray photons or other photons as the case may be.

The beauty of continuously improving the precision of scientific experiments is revealed by the recent work on phonons. Since these are bosons, they should exhibit boson condensation. This would be the equivalent of laser radiation for photons and superfluids for composite bosons. These have now been experimentally realized. In the discussion on photons as representative bosons, it was pointed out that laser radiation is a quantum wave which obeys an uncertainty relation linking the phase and number of photons and that squeezed states are realized when the uncertainty in phase and number is not equal. Similarly, enhanced accuracy for the phase of the wave is obtained by increasing, deliberately, the uncertainty in the knowledge of the number of phonons present. Squeezed phonon states have been demonstrated to result in a temporary reduction in the amplitude of vibration of the atoms in crystal lattice. The quantum nature of the phonon has thus been confirmed to a degree equivalent to that of the photon.

Magnons and magnetic order

The magnetic moment associated with an atom is the sum of the contributions from the movement of an electron in the orbital and its intrinsic spin. Many properties of condensed matter were explained by considering the properties of a collection of “free or non-interacting spins”. Some materials however exhibit macroscopic magnetic order which involves

broken symmetry and the emergence of a quasi-particle called the magnon. The entire piece of iron for example, exhibits a net magnetic moment. From the magnitude it appears that magnetic moments associated with all the ~10^{23} iron atoms, are aligned in one direction. Several such long range alignments shown in Figure III.21 are experimentally confirmed. Of the three types of order, ferromagnetic and ferrimagnetic order are exhibited by metals, (iron, cobalt and nickel) in addition to several of their oxides. Anti-ferromagnetic order can be deduced from measurement of resistance or heat capacity of metallic chromium and nickel oxide. The estimated internal magnetic field responsible for the observed alignment is at least one order of magnitude larger than the highest magnetic fields achieved in the laboratory and is too large to be attributed to magnetic interaction between atomic magnetic moments. Ferroelectric materials which exhibit a permanent electric field rather than a permanent magnetic field are experimentally known. In contrast to ferromagnetic materials, these are described completely by the forces between electric dipoles. This is another instance of the electric forces being much stronger than magnetic forces.

The strong magnetic ordering is another consequence of Pauli Exclusion Principle. As discussed for atomic orbitals, due to Pauli Exclusion, electrons with parallel spins are separated in space. However this separation reduces coulomb repulsion between the electrons. Ferromagnetic ordering is favored when rather than anti-parallel spins in close proximity, parallel spins separated in space have lower energy. This is called exchange since

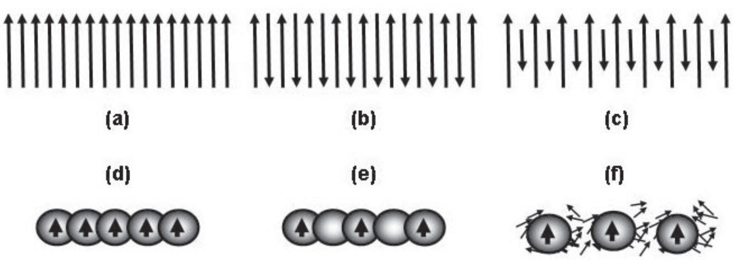


Figure III.21. The three types of magnetic order (a) Ferromagnetic, (b) Ferrimagnetic and (c) Anti-ferromagnetic along with the three types of exchange (d)Direct exchange, (e) Super exchange and (f) Indirect exchange

an anti-parallel spin is exchanged for a parallel one. Exchange energy J is comparable to the difference in energy between the parallel spin (triplet ➔ ↑↑) and anti-parallel (singlet ➔ ↑↓) spin states. The large exchange energy is responsible for the strength of magnetic ordering. The singlet state has lower energy in a helium atom while the triplet state has lower energy in a ferromagnetic material. This is called direct exchange. When the exchange involves an additional non-magnetic ion it is called super-exchange. Both these are observed in insulating magnetic materials such as ferrites. For explaining the magnetism of metals (iron, nickel, cobalt and their alloys) it is necessary to consider the exchange indirectly mediated by the conduction electrons. The three exchange schemes are also represented in Figure III.21.

It is obvious that in a ferromagnet, isotropic order of the spins is completely lost and there is long range ordered alignment of the spins. Thus the formation of new quasi-particles is to be expected. The new excitations are called magnons. Just as phonons can be visualized as oscillations of atomic position, magnons are visualized as long range oscillations of spin directions as shown in Figure III.22 (a). A bulk magnetic material does not normally show a net magnetic field in its vicinity. All pieces of iron can become magnetic but are quite often not magnets. A ferromagnetic material without a net external magnetic field consists of domains. Domains are macroscopic regions where all spins are aligned

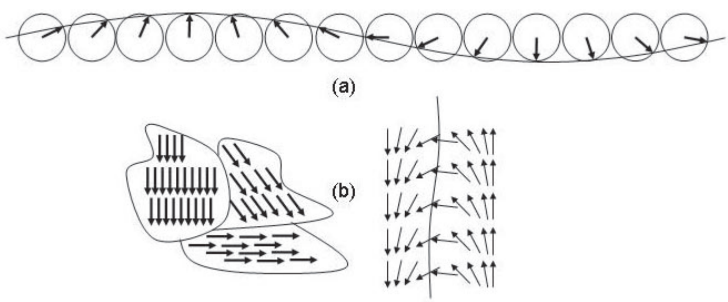


Figure III.22 The magnon represented (a) As a wave of the spin direction and (b) As the variation in spin direction at the domain boundary.

and there is a net magnetic field. The magnetic fields from various domains cancel resulting in the net external field being zero as shown in Figure III.22 (b). Atomic magnetic moments inside a domain are aligned and this can be confirmed experimentally. The alignment of magnetic moments in the domain boundaries resembles the spin wave discussed above. While the dipole-dipole interaction is not capable of aligning atomic spins inside a domain, these are responsible for the formation of domains in a macroscopic body, their orientation and growth under the influence of an external field. Existence of magnons, the quantized spin oscillations, is confirmed by neutron scattering. Neutrons can interact with the spins since the neutral particles are not subject to the coulomb forces that influence electrons. Recently condensation of magnons has also been experimentally observed, confirming the basic quantum nature of magnons.

Other quantum particles

A veritable zoo of particles has been identified in condensed matter systems. Some of the more prominent, apart from phonons and magnons are excitons, polarons, plasmons, polaritons, rotons and “Cooper pairs”. While the Landau spontaneous symmetry breaking is a powerful theoretical unification of results, formation of many of these quasi-particles are conjectured in the spirit of Einstein’s extension of Planck’s formula for lattice vibrations.

Plasmon: A metal consists of free electrons and the positively charged lattice of the nucleus along with the core electrons. This constitutes plasma, a fluid which consists of charged particles. Formation of waves in such a fluid is an obvious possibility. Therefore extending the idea of quantization of waves, it is a straightforward generalization to postulate the existence of plasmons (quantized oscillations in the plasma). Experiments can then be performed to detect the plasmons and verify that they behave as quantum objects obeying Planck’s law. Not only plasmons propagating in the three dimensional metal but also surface plasmons, confined to the boundary between a dielectric and a thin metal layer have been detected. The latter are extensively used for bio-sensor applications. In order to

confirm the quantum nature of these oscillations, and their boson character, efforts are made to condense these into a Bose condensate. Signature of such condensation are claimed to have been obtained. Practical devices using plasmons rather than electrons to convey information in computer chips are also in progress.

Exciton: In a semiconductor, presence of free electrons and holes is well established. A coulomb attraction between the two and the formation of a combined particle like a hydrogen atom is energetically possible. The energy is however very small, usually a few tens of milli-electron volts compared to approximately 10 electron volts for the hydrogen atom. But if the experiments are performed carefully, the existence of these "quasi-hydrogen atoms" inside a semiconductor should be detected. Experimentally not only the formation but also the excitation of these excitons to higher energy levels precisely like the excited states of hydrogen can be detected. Obviously Bohr's quantization of the electron momentum is a useful concept here. Like the helium atom, the exciton must be a boson and should exhibit condensation. Not very surprisingly, these have been observed. Typically the exciton is much larger han a hydrogen atom but can be confined by proper selelction of the size of the semiconductor. Thus the exciton has been used ina practical device to be described in the next chapter.

Polaron: Polaron is an entity quite similar to an exciton encountered most often in a conducting polymers such as poly-aniline, poly-acetylene or poly-thiophene formed with alternate single and double bonds between the monomers. Here, the charge (electron or hole) is created by introducing a defect. An oxidizing or reducing chemical agent is employed to either add or remove an electron from the chain. The defect creation also results in a distortion of the polymer and consequently, polarization of the polymer as shown in Figure III.23. The movement of the charge will then be accompanied by the movement of this polarization. The combination behaves like a single quantum mechanical particle, "the polaron" with distinct energy levels, which, as in the case of the exciton, are in the forbidden band (i.e.,the states are localized). When the interaction is between a photon and a distortion in the lattice, a particle called a "polariton" is

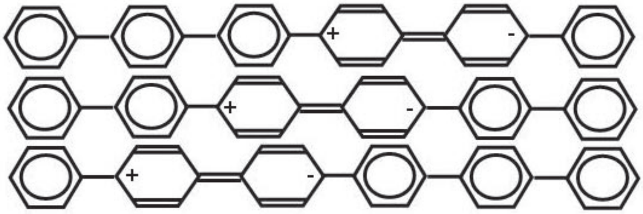


Figure III.23 Schematic of polaron formation on a conducting polymer.

postulated. While a single polaron is a fermion, a pair of them can bind together and form a bipolaron without net spin (as in the case of a pair of electrons). Extremely sophisticated theoretical modeling of these systems is extensively cited in literature. For the purpose of the present discussion, the important question is the extent to which the quantum nature of these entities has been experimentally established. Quantized energy levels, which confirm their quantum nature and tunneling through a barrier, have been observed, establishing the distinctly quantum mechanical nature of these particles. Condensation of the bosons is also invoked as an explanation for some experimental results.

Cooper Pair: It is quite difficult to identify the symmetry that has been broken resulting in the formation of particles such as the exciton and the polaron. It is also not very useful.The formation of Cooper Pairs in superconductors is much more easily described in the language of broken symmetry. A superconductor is a material that exhibits zero resistance and perfect diamagnetism below a transition temperature. The absence of resistance in a real macroscopic material is extremely interesting. The presence of defects, impurities etc. do not seem to effect the situation. As in the case of the superfluid, it has been experimentally demonstrated that the period over which the flow of electrons in a closed superconducting ring would decay is comparable to the known life of the universe, several billion years. Zero resistance of a superconductor is an indication of a Bose condensed state just like the zero viscosity of a superfluid. The most important characteristic of a superconductor is its perfect diamagnetism. An external magnetic field is expelled from within the superconductor

(Meissner effect). This is equivalent to the breaking of the gauge symmetry of electromagnetic field. Quasi-particles called "Cooper Pairs", consisting of two electrons, subjected to an attractive force, due to the mediation of a phonon, form. The new long range order however is only obvious in the momentum space. The Cooper Pairs being bosons condense into a single electronic state at low temperatures. In a descriptive language, the attractive force between the electron and the lattice causes a deformation of the lattice, which then causes the second electron to move towards the first due to the attraction between that electron and the lattice as shown in Figure III.24.

All the Cooper Pairs in a macroscopic superconducting body condense into the single lowest energy level since they are bosons. When a thin potential energy barrier exists, the wave function of a quantum object can have finite values both within the barrier and beyond. Thus there is a probability for the particle to tunnel through this region. This situation which has no classical counterpart has been mentioned above in connection with phonons and other quantum particles. The tunneling of Cooper Pairs through a thin film of an insulator between two superconductors is called the Josephson Effect. Tunneling is another uniquely quantum phenomenon. When there is a barrier to the movement of a quantum particle, for example an insulator between two metals, quantum mechanically, one can only state that the probability for the presence of the electron inside the barrier is very small. If the barrier is very thin, the probability becomes large

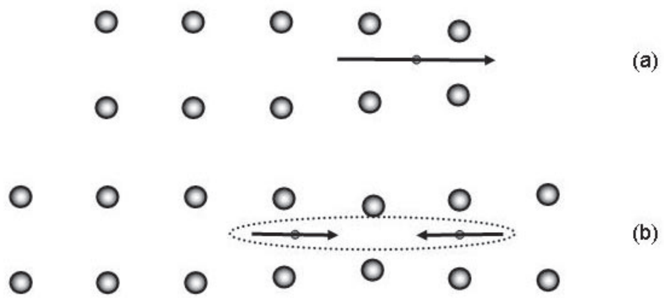


Figure III.24. A descriptive account of Cooper pair formation (a) The moving electron distorts the lattice. (b) The distortion causes an apparent attractive force between two electrons.

enough for the quantum object to be observed to have passed through the barrier. It has been experimentally confirmed that the pair of electrons tunnel together through the barrier. Various other aspects of the quantum nature of the Cooper Pair have been confirmed experimentally. With the exception of the electron and the photon, no other quantum object has been as extensively investigated as the Cooper Pair.

It has been possible to make the two electrons constituting the Cooper Pair travel along two separate conductors which are placed close to one another. Still the basic boson character of the Cooper Pair is not lost. When a thin insulator barrier is introduced in a superconducting ring, interference between macroscopically large super currents can be observed. It has also been possible to monitor the movement of individual Cooper Pairs into a container. The fact that a combination, of a quantized lattice vibration and two individual electrons, behaves like a new quantum object, with significantly different properties, is important in understanding quantum nature.

Roton: The quantum objects encountered in a superfluid are even more exotic than the Cooper Pair. As was mentioned earlier, the helium atom, a collection of six fermions behaves like a boson and exhibits superfluidity. A new quantized collective motion of the helium atoms called a "roton" exists in addition to phonons. It was mentioned at a superfluid, once in motion never stops. Strictly speaking, this is true only when the velocity is smaller than a critical value. Above the critical value of the velocity, the superfluid slows down due to the creation of rotons. The existence and quantum nature of rotons has been experimentally confirmed. A compact volume of the superfluid cannot rotate. Vortex lines (i.e., holes) in the superfluid have been observed around which the superfluid rotates. The circulation of the velocity around the vortex lines is quantized in multiples of h/m. Vortices have also been observed in superconductors. In some superconductors (called type II), there is an intermediate state between the normal and superconducting states. Here the magnetic field penetrates into the superconductor but is confined in small normal regions. These regions are surrounded by superconducting regions in which super currents

flow around the normal regions. In view of the quantum nature of these super currents, the magnetic flux confined in these regions is a multiple of a basic flux quantum called the fluxon. This has a value of $ö_0 = h/2e = 2\times10^{-15}$ weber and this quantization is exploited in a device for measuring magnetic fields with incredible accuracy as will be discussed in the next chapter.

The effect of dimensionality

The discussion so far has shown the emergence of newer quantum particles and exceedingly interesting phenomena in collections of photons and electrons. A key aspect of this collective behaviour is the dimensionality of the system which is three. Recently experiments in which electrons are confined to two dimensions have been performed. The theoretical and experimental results have shown the complexities that emerge at lower dimensions. The experiments are performed using GaAs/AlGaAs High Electron Mobility Transistors (HEMT). The details of this device will be discussed in the next chapter along with various other devices designed

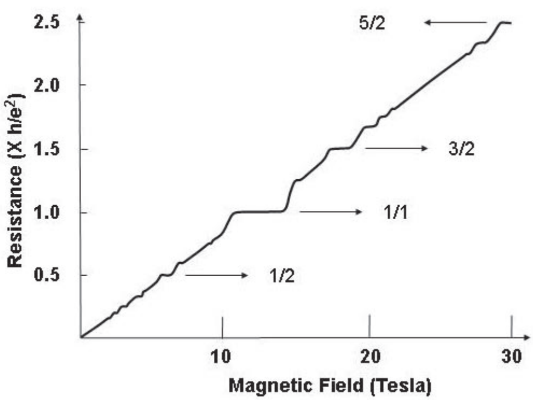


Figure III. 25. The variation of the Hall resistance of HEMT device showing plateaus at integral and fractional multiples of h/e^2. (Adapted from Yasuhiro Iye, "Composite fermions and bosons: An invitation to electron masquerade in Quantum Hall", Proc. Natl. Acad. Sci. USA 96, 8821,1999.)

using quantum ideas. For the present, it is only necessary to state that the electrons in this transistor are confined to a two dimensional surface between GaAs and AlGaAs semiconducting layers. Interesting physics emerges when the device is subjected to a very high magnetic field, 30 tesla or approximately 300 kilogauss. In comparison, the magnetic field of the earth is ~0.3 gauss.

The resistance of a material when subjected to a perpendicular magnetic field is called the Hall Resistance, and these measurements are routinely carried out on semiconductors to determine the type of carriers (electrons or holes) and their numbers. Since the Hall voltage is inversely proportional to the number of carriers this is a very useful method for semiconductor characterization where the number of carriers are low. The HEMT shows an extremely interesting variation of the Hall resistance with the magnetic field. Figure III.25 shows the results, where the resistance is plotted in units of h/e^2. Plateaus, (regions where the resistance does not change as the magnetic field is varied) are observed at integral multiples of h/e^2 and some simple fractions such as 1/2, 3/2, 5/2 which are highlighted in the figure. The integer plateaus are more prominent (persist over a larger range of the magnetic field).

It is intriguing that an experimentally determined quantity such as resistance can have a universal value independent of the sample. Resistance of a real object is expected to be dependent on its size, temperature and processing conditions. In this context to find the resistance to be exactly equal to the ratio of the Planck's constant and square of the electron charge (both universal constants) is extremely surprising. Not even resistivities of single crystals of simple metals are calculable from theory. In contrast, these universal values are experimentally well established to a very high accuracy and have now been accepted as the primary standard for resistance. The physics of this phenomenon is extremely relevant for the discussion of the present chapter, on the consequences of collecting a large number of quantum particles as is inevitable in a sample of macroscopic condensed matter.

The distinction between fermions and bosons is related to the particle exchange as discussed at the start of the present chapter. As will be discussed in the next chapter where the details of the HEMT are provided, the electrons are confined to two dimensions. During the exchange of particles in two dimensions, the "clockwise and anti-clockwise exchanges" shown in Figure III.26(a) are distinct. In contrast, in three dimensions, the two are identical since it is possible to continuously move from one to the other using the third dimension. In view of this distinction, in two dimensions, the fermion is converted to a boson and vice versa when it is attached to one or more quantum of fluxes. The flux quantum (2×10^{-15} weber) has already been encountered in the earlier discussion on Josephson Effect. When an odd number of flux quanta, each with a flux h/2e are attached to an electron, as shown in the figure III.26(b) and (c), during exchange, the electron behaves like a boson. On the other hand if the number of fluxons is even, the fermion and boson characters are not altered. In three dimensions, as was discussed in several examples, two fermions with opposite spin together act like a boson.

To understand the Hall measurement results mentioned above, the number of electrons that can exist in a given energy level in this two

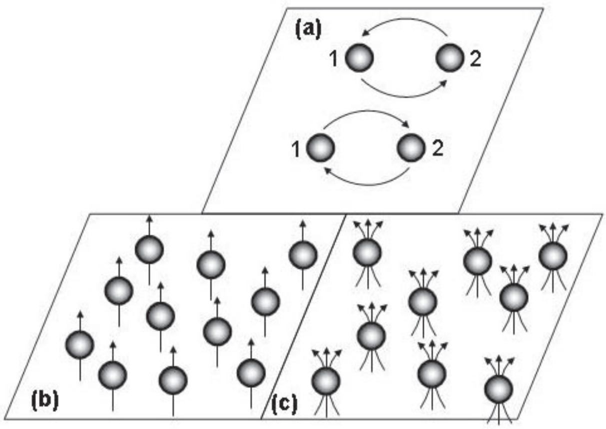


Figure III.26. (a) In two dimensions, the clockwise and anti-clockwise exchanges are distinct. In (b) and (c), the electrons are associated with one and three fluxons and thus converted into bosons.

dimensional system has to be considered. Firstly, a large magnetic field aligns the electron spin in the direction of the field. Secondly, the electron rotates around the magnetic lines of force due to the familiar Lorentz force. The orbits of the electrons can take only those states which are integral multiples of the electron wavelength. This is equivalent to Bohr's quantization of the orbits of the electrons in a hydrogen atom. Each electron repels all the others and it should be noted that the coulomb forces of repulsion are long range forces, which have significant values even at large distances of separation between charges.

If the number of electrons present is large, electrons are forced to approach each other. The orbits of the electrons around the lines of force have to become smaller. Correspondingly, the electron wavelength has to become smaller which in turn means that the electron energy is higher. Thus, as the number of electrons increases, all of them cannot be accommodated in the lowest energy level, and they are accommodated in the higher energy levels. These levels are called Landau levels and the number of electrons that can be accommodated, per unit area, in a given energy level, can be calculated. Thus, given a number of electrons, some of the levels would be occupied. As the magnetic field is increased, the number of levels occupied will decrease and eventually all electrons will be accommodated in the lowest level. While the mathematics is extremely complex, it is interesting to note that the states of the electron gas can be described as emerging from a state of composite fermions with an odd number of fluxons attached to it. The state with one unit of flux corresponds to the resistance of h/e^2 and that with three fluxes to a state of resistance 1/3 of this value. However, it is equally possible to consider the electron coupled to three fluxons as a boson and then view the quantum hall energy states as superfluid state of the bosons.

Other quantum possibilities

In qualitatively describing the collective behaviour of quantum objects, it was pointed out that several physical properties of macroscopic condensed matter can be understood ignoring interactions in the collection

of quantum objects such as spins and electrons. Several instances of collection of fermions behaving like bosons were explored with macroscopic quantum condensed states being observed in some cases. Dimensionality plays a crucial role in such systems. In several other cases, the interactions between quantum particles cannot be ignored. The Landau theory of symmetry breaking helps in analyzing such systems in terms of new quantum quasi-particles which represent collective excitations of these interacting quantum particles. The foregoing may lead to a mistaken belief that all aspects of collective behaviour are now completely understood. To avoid this, in this section two aspects are discussed where the framework developed so far is not quite satisfactory.

The first of these is the behaviour of the systems close to the temperature at which symmetry is broken. Treating the collective behaviour as a quasi-particle is sometimes technically referred to as a mean field description. The magnon description of a ferromagnet is an example. In study of ferromagnetism, the internal field invoked as the cause for the alignment of the spins explains the name. This is a mean or average effect of all the spins acting on any single spin. The mean field description is not very satisfactory when materials are probed very close to the transition temperature (also called the critical point) at which the symmetry is broken. Properties, such as magnetization, electrical resistance and specific heat are measured at reduced temperatures (defined as $(T-T_c)/T_c$ where T_c is the transition temperature) of 10^{-6}. The mean field theories make precise quantitative predictions of the variation of the physical properties as a function of the reduced temperature. Measurements show distinct deviations from the mean field predictions.

These studies also revealed the dominant role of dimensionality in the physics near phase transitions. Superfluids, superconductors, ferromagnets and anti-ferromagnets are examples of systems discussed here which exhibit critical points. There are many other systems including partially miscible liquids and water at high temperature and pressure. Many systems show identical behaviour very close to the critical point, ignoring the detailed physical origin of the order. For example, all liquid–gas systems

show identical behaviour, independent of the chemical composition of the fluid and exactly match the ferromagnetic phase transition in uniaxial magnets. It was observed that all systems exhibiting a critical point can be characterized by two integers (1) The dimensionality and (2) The number of components of the "order parameter". A classical theory has been developed to make predictions which have been confirmed with incredible accuracy. While discussing this enchanting physics would take the present discussion too far, it is important to note the role of the dimensionality and that a classical description emerges independent of the precise quantum description of the interaction responsible for the order.

There are also very recent materials where the Landau symmetry breaking approaches are not supported by experiments but no satisfactory alternate theory has so far emerged. The important systems include heavy fermion systems, high temperature superconductors and materials exhibiting giant and colossal change in resistance with applied magnetic field. There is no real consensus on an approach for understanding these systems. It is generally accepted that correlations and interactions being extremely strong is responsible for the breakdown. Intriguing ideas such as separation of charge and spin are being seriously discussed. At least in one dimension, as shown in Figure III.27, the separation of spin and charge can be simply visualized. The picture shows a vacancy in one dimensional chain where the neighboring charged particles show anti-ferromagnetic order. A vacancy

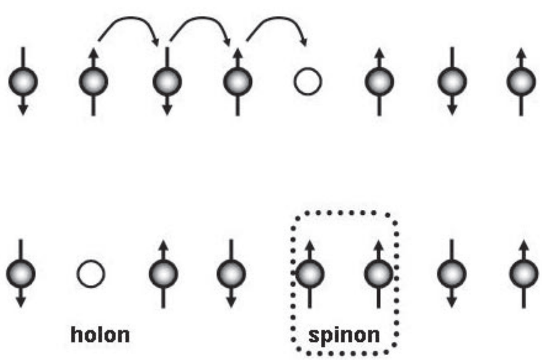


Figure III.27. Schematic of charge spin separation in one dimension.

permits conduction and is termed a “holon” which is a charged object. Movement of the defect leads to a new particle “spinon” which has no charge associated with it. Many more interesting quantum objects are likely to be hypothesized and experimentally confirmed.

IV
Designing Quantum Devices

Chapter I provided a basic description of the quantum theory. The second discussed experiments which leave the scientist with no alternative except to accept that nature has to be described in terms of quantum entities though their behaviour is counter-intuitive. The third chapter analyzed the consequences of collecting a large number of quantum particles and how the collective behaviour of these particles explains observed phenomena, ranging from optically transparent glass to a superfluid which has zero viscosity. At several points, the quantitative strength of the quantum theory has been stressed. The purpose of this chapter is to discuss several devices and technologies that are based explicitly on quantum theory.

Since quantum theory is the basis of all of physics, chemistry and biology, any commercial device is an example of quantum technology. In the present chapter however, devices which have been developed after the quantum theory had become well established, that is after 1940, are discussed. The examples considered here were entirely designed with a prior understanding of the quantum physics that permits such a device to be constructed. Some of these devices were developed by trial and error. Even when the initial experiments have been done for some other reason, the scientists and technologists have immediately caught the quantum significance of the results and have immediately fine tuned the devices on that basis. Most of the devices discussed are now commercially available. In a few cases, ideas which are now being implemented in the laboratory have been included, in view of their future potential. The entire quantum

physics being discussed in terms of electrons and photons, it is no surprise that the devices are almost entirely electronic or optical in nature.

The transistor

The first device to be described is the transistor. By all accounts it is the "defining" invention of the modern age. It started the electronics era and is the basis of all modern technology. Progress is being achieved through the development of smaller, better or faster transistors. No new building block, capable of replacing the transistor principle, has been identified so far. The motivation for the development of a transistor was to find an alternative to the triode, a vacuum tube with three contacts. The triode consists of a source of electrons such as a heated filament (cathode), a plate (anode) attracting the electrons with the help of a high electric voltage and a grid between the two through which the electrons pass. The entire assembly is housed inside an evacuated glass tube very much like an incandescent bulb. The electric current between the cathode and the anode can be altered by changing the voltage applied to a grid. A large current in the output can be changed by changing a small voltage in the input. This amplification capability in a triode was used to build the first computers. When large numbers of these triodes were used in the computers, their reliability was not satisfactory and the power consumption was too large. A better amplifier device was clearly desirable.

The transistor was envisaged as a new three terminal device with a current amplification capability. The first device as is well known was built in 1947 and consisted of two closely spaced gold point contacts made to a semiconducting germanium crystal. Injecting a small current through one of the gold contacts (called the base contact) changed the current between the semiconductor and the other gold contact (called the emitter and the collector) by a large amount, producing a current gain. Three years later a second device called the junction transistor was made, where the gold contacts were replaced by forming junctions between doped

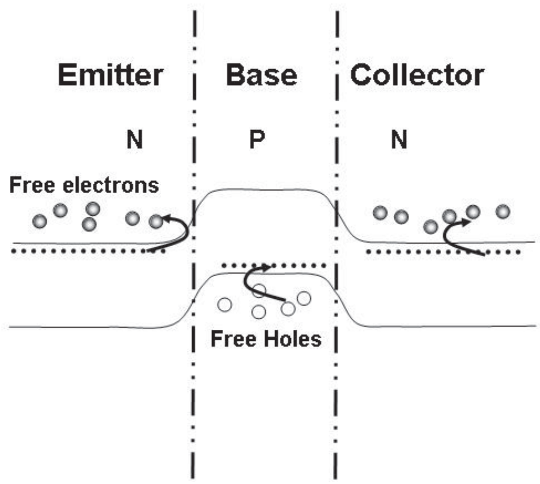


Figure IV.1 The band diagram of NPN transistor.

semiconductors. Once again, a small change in the base current results in a large change in the emitter collector current.

The NPN transistor shown in the Figure IV.1 consists of two layers of n type semiconductor acting as the emitter and collector separated by a thin base region of the p type semiconductor. As mentioned in Chapter III, electron (n) and hole (p) conduction is achieved by doping the semiconductors. The types can be reversed to obtain a PNP transistor. The current between the emitter and the collector is controlled by injecting a small current in the base region. The narrow width of the base is necessary to ensure that most of the carriers from the emitter are not being removed at the base but travel into the collector region. Clearly, the bending of the bands in the transistor is identical to that in the p-n junction discussed in chapter III. The base acts as a common contact for the two p-n junctions. The formation of a junction potential and depletion width discussed for the p-n junction are applicable to the two junctions in the transistor.

Consider the quantum ideas behind the design of a transistor. Firstly, the selection of Si and Ge as suitable materials is based on quantum

theory of electronic conduction in semiconductors. Only these two elemental materials have reasonable band gaps so that it is possible to dope the materials with other elements and get reasonable concentrations of free electrons and holes. Even fifty years later, forming a transistor using diamond is extremely difficult even though carbon, like silicon is an element with four electrons in the outer most shell. Secondly the scientists at the Bell Laboratories, where this work has been done also knew about the hole and electron conduction in doped semiconductors. The p-n junction formation was confirmed before a third contact was introduced for current control.

When the first transistor devices were built they did not show any amplifier action. The failure was attributed to surface states. Just as doping phosphorus in silicon leads to defect levels in the forbidden energy band, close to the conduction band in an "n" type semiconductor, the surface introduces defect states in the semiconductor. The atoms on the surface have silicon atoms on one side but not on the other side. The surface defects however do not benefit the operation of the junction by creating extra free carriers but screen (reduce) the voltage in the interior of the semiconductor. Thus the theoretical concept of localized surface states was used to understand the failures. Following this understanding, the device was made operational by submerging it in a liquid which reduced the problem due to the surface states.

As mentioned above, a narrow base is critical for functioning of a transistor. The transistor has been designed with the "band picture of electron states" identifying the role of a narrow base region. This alone ensures that the electrons from the n type emitter region can directly flow into the n type collector region without significant recombination with the holes in the p type base. The junction transistor was designed and patented immdediately following the demonstration of the point contact transistor with a clear theoretical understanding of carrier transport. All this shows that the transistor could be designed only when the quantum theory was well understood.

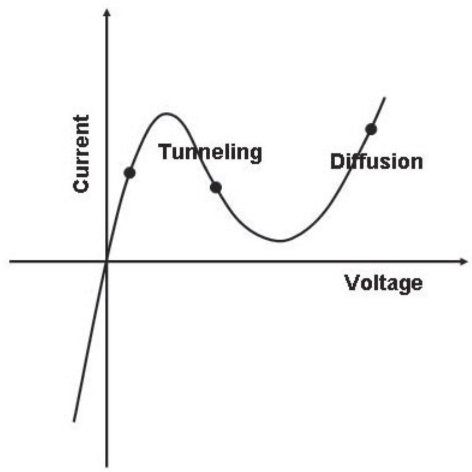


Figure IV.2. The current in a tunnel diode decreases as the voltage is increased in a small region demonstrating negative resistance.

The tunnel diode

The tunnel diode or Esaki diode exhibits negative resistance. As the voltage on the device is increased, the current instead of increasing begins to decrease, and exhibits a minimum. The initial observation was made while investigating the failure of the newly developed 2T7 transistor for wireless receiver sets at the Sony Corporation. Phosphorus doping in the base region of the PNP transistor was increased in an attempt to create a better transistor. Instead, the device failed to function properly. Subsequent measurements confirmed the peculiar form of the current voltage characteristic shown in Figure IV.2. Knowledge of quantum theory immediately enabled Esaki to identify the quantum processes responsible for the peculiar behaviour. When heavily doped, the depletion width of the p-n junction (The region at the p-n junction, where the free carriers from one type of semiconductor diffuse into the other type leading to an electric field) is reduced. The possible reduction in depletion width was the reason for trying to increase phosphorus doping in the transistor. The small value of the depletion width permits both tunneling of carriers and diffusion of nearly free particles. Tunneling was mentioned several times in

the earlier chapters and discussed when the Josephson Junctions were introduced in the last chapter. Normally, in a p-n junction, the electrons and holes can only move over the barrier when an external voltage is applied. Thus, the p-n junction acts as a rectifier. When the voltage is reversed, instead of assisting the movement of carriers, it prevents their motion. Current flow in one direction is much smaller than in the other. Tunneling makes the situation more complex. Because the depletion width is small, the electrons in the conduction band of the n-type semiconductor have a finite probability for being present on the p-side. The same is applicable for the holes in the p-type semiconductor. Thus the electrons and holes can tunnel into the other side if there are unoccupied energy states in the other side.

The conduction processes at various external voltages are shown in Figure IV.3. At zero or negative voltages (points a and b), a large number of electrons from the heavily doped n region (as also the large number of holes from the heavily doped p region) can tunnel across. As the positive voltage is increased (point c), the holes cannot tunnel since the band has moved as shown in figure and the holes at the top of the valence band

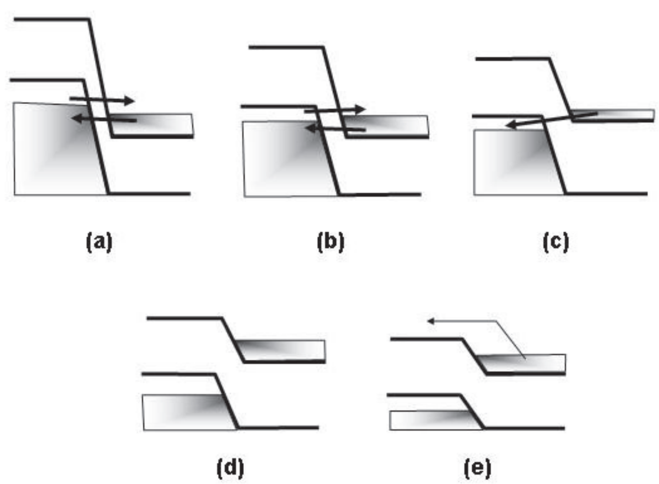


Figure IV.3. Change in the band picture of a tunnel diode as the applied voltage is increased from (a) to (e).

have no unoccupied states with suitable energy in the n-type material. This region is marked as "tunneling" in Figure IV.2. As the voltage is increased further (point d), electron tunneling also ceases, leading to lowered current (the minimum in Figure IV.2). At very large voltages, (point e) the operation is similar to that in any ordinary p-n junction. This is marked as "diffusion" in Figure IV.2.

The quantum mechanical understanding enabled optimization of the device for various applications. The tunnel diode is a low current device but operates at high frequencies since the time required for tunneling is much smaller than for the normal diffusion or drift of carriers. The device is extensively used for high speed switching and oscillator circuits. As is well known, the electronics revolution is based on extremely small transistors. During designing these, care has to be taken to ensure that tunneling does not degrade device performance. It is also possible to design new electronic devices which exploit tunneling mechanism. One example is considered next.

Resonant tunneling transistor

The resonant tunneling transistor is a recent device design that exploits several quantum mechanical ideas. While the commercial future of the device is still an open question, the design provides some interesting use of basic quantum mechanical approximations. The first discussion in any quantum mechanics text book is labeled "the particle in a box". Basically the quantum particle is assumed to be localized in a region of low potential energy and the walls of the "box" are regions of high potential energy. A classical analogue would be a ball floating in a deep well. The ball, a classical object can only be taken out when energy (equal to the difference in potential energy between the top and bottom of the well) is provided to it, but as we have observed, the quantum objects do not obey common sense rules. A "quantum particle" in a box can tunnel out of the walls without external energy and can have only discrete specific energy values as it is raised to the top of the potential well. This is the most idealized model that can ever be imagined. Simple mathematics then enables

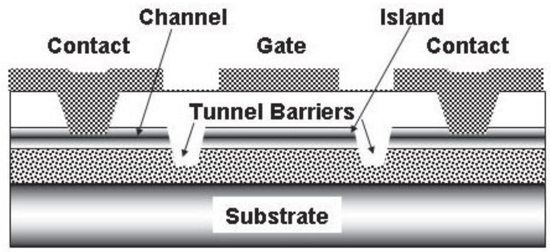


Figure IV.4 Schematic view of a resonant tunneling transistor.

a student to derive the energy values permitted for the quantum particle in the box. As usual, due to the periodic nature of the amplitude or wave function, the particle does not have a probability for being in any other energy level.

The resonant tunneling transistor is an actual experimental realization of the particle in a box. The device is schematically shown in Figure IV.4. A small region of the conducting channel, 10-100 nm in size is converted into an island by the use of two tunnel barriers. The small size of the island ensures that the energy levels are discrete levels as for a particle in a potential well. The only difference form the text book example

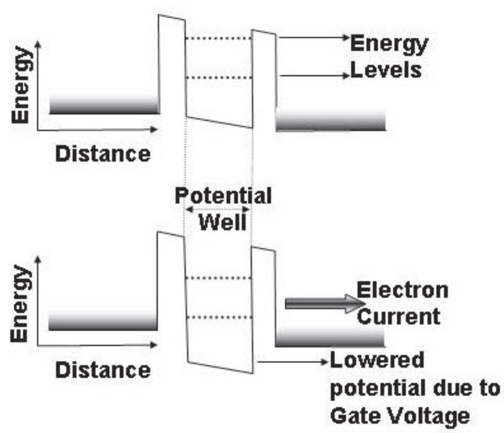


Figure IV.5 Energy level diagram of a resonant tunneling transistor with and without gate voltage.

is the narrow width of the barriers (the walls of the box) which permits electron tunneling. The rest of the channel is relatively large and contains many free electrons in the conduction band. A gate is provided to change the potential in the box (island). Electrical contacts are made to the channel on both sides of the island.

The electron energy levels in the device are shown in Figure IV.5. The lowest energy level in the potential well (island) is designed to be higher than the energy levels of the electrons in the conduction band of the channel. This is the basic quantum mechanical design of the device. Thus, the island and the two tunnel barriers together act as a single barrier. The size is too large for significant tunneling of carriers and there is no current between the two metal contacts. However, when an external voltage is applied, the potential inside the island is lowered. At a specific voltage, the discrete levels in the well are aligned with the electron energy in the conduction band as shown in Figure IV.5. The island is no longer a part of the barrier to tunneling. The two small barriers now permit the tunneling of electrons from the channel into the island and out of the island into the second part of the channel. This results in a high current in the device between the two contacts. The device has not been designed to experimentally prove quantum mechanical description of a particle in a box. It is an example of advanced electronic device which may be commercially used in future since the design permits operation at higher speeds and the performance does not deteriorate if the dimensions are reduced.

High Energy Mobility Transistor (HEMT)

The HEMT is another device that originated in empirical efforts to optimize commercial devices but the initial surprising results were rapidly understood on the basis of quantum theory. The problem was the perennial failure of GaAs devices to compete with silicon. With a direct bandgap and higher mobility, GaAs was expected to replace silicon in advanced semiconductor devices but this has continued to be an unrealized potential. The reason is SiO_2 (an insulator) that can be grown on silicon with minimum

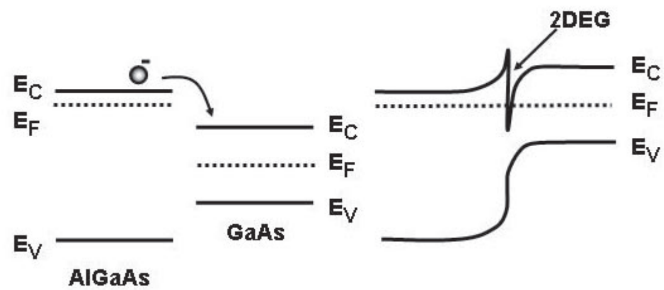


Figure IV.6 Band diagram of a junction between AlGaAs and GaAs layers.

localized or surface defects. GaAs does not have a suitable dielectric to challenge the Si/SiO_2 combination. It was proposed to completely eliminate this problem with a wide bandgap AlGaAs layer. Since this is a single crystal layer, just like GaAs, with a matching unit cell size, it can be grown without a crystal boundary and the consequent localized interface (defect) states. The device could then be expected to work more satisfactorily and provide a high frequency transistor. While the expensive epitaxial growth process, used for growing single crystalline layers of GaAs and AlGaAs was not expected to challenge low cost silicon processing at low frequencies, it should enable GaAs to occupy a niche market in microwave and high frequency applications. The result was however surprising. The electrons from the n-doped GaAs, cross the boundary to the wide band gap AlGaAs, but are localized in a region of ~10 nm near the interface. The band diagram is shown in Figure IV.6. The de Broglie electron wavelength is much larger than 100 nm. Thus the electrons are confined into two dimensions and labeled as 2 DEG (Two Dimensional Electron Gas). The lack of scattering centers in this region ensured that the electron mobility is very high and that the original requirement of a high quality transistor is achieved but not operating like a MOSFET as originally envisaged. (The Metal Oxide Semiconductor Field Effect Transistor (MOSFET) operates very much like the resonant tunneling transistor described above. The voltage on the gate controls the current between the two contacts called the source and the drain). The schematic view of an actual HEMT device is shown in Figure IV.7. The successive layers are all single crystalline GaAs with suitable additives and are grown before

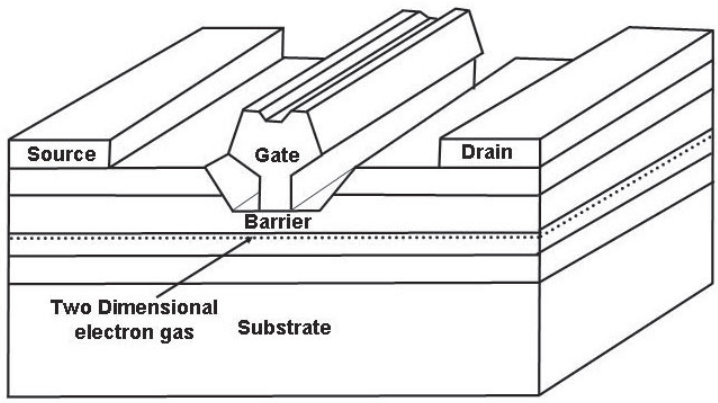


Figure IV.7 Schematic view of the HEMT device.

lithographically separating the regions and metal contacts are deposited. The HEMT devices are extensively used in many high frequency communication systems.

The behaviour of quantum particles in two dimensions was described in Chapter III. The discovery of Integral Quantum Hall Effect (IQHE) resulted in Klitzing being awarded a Nobel Prize in Physics in 1985 and the discovery and explanation of the Fractional Quantum Hall Effect (FQHE) resulted in a Nobel Prize for Laughlin, Stomer and Tsui in 1998. Two dimensional behaviour is also connected to other puzzling phenomenon such high temperature superconductivity in complex oxides. These materials are superconducting at temperatures as high as -100ºC. The HEMT is an example of symbiotic development of technology and fundamental physics. As mentioned before, the conversion of fermions into bosons in two dimensions is different from an atom of helium where six fermions in three dimensions jointly behave like a boson. Practical utilization of a device, which exploits the properties of electrons in two dimensions, will be discussed later in Chapter VI when the philosophical issues are taken up briefly.

Quantum well optical modulator

This device exploits the current engineering ability of building quantum mechanical potential wells in reality. In that sense it is another

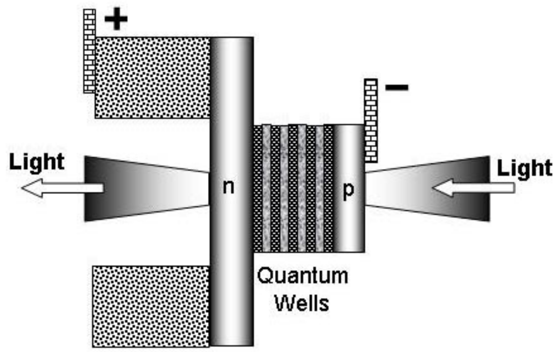


Figure IV.8 Schematic view of a quantum well optical modulator.

quantum design, just like the resonant tunneling transistor, discussed earlier. This device exploits the quantum mechanical properties of a composite quasi-particle, the exciton. The device is schematically shown in Figure IV.8. This is another device built using single crystalline layers of GaAs and AlGaAs. It is a two terminal device and the operation is very simple. It is normally transparent to light of a particular wavelength but by applying a specified voltage to the two electrical terminals, the device can be made completely opaque. The key technological utility is the speed with which this can be accomplished. Operating in the GHz region, this device is used for modulation in optical communication.

To understand the process of designing this device the concept of an exciton has to be used. Excitons in bulk semiconductors are quasi-particles, formed by the binding of an electron in the conduction band and a hole in the valence band. The binding energy is significantly lower than the thermal energy at room temperature and the exciton decays very rapidly with the recombination of the electron and hole. This releases, in the form of a photon, the energy originally supplied to move an electron, from the valence band to the conduction band to create the hole. An electric field can be used to cause exciton dissociation, since the electron and the hole move in opposite directions. The electron hole pair then recombines to release the energy in an external resistance. This is the principle of a solar cell, in which the energy for creating the electron hole pair is provided by the photon from sunlight.

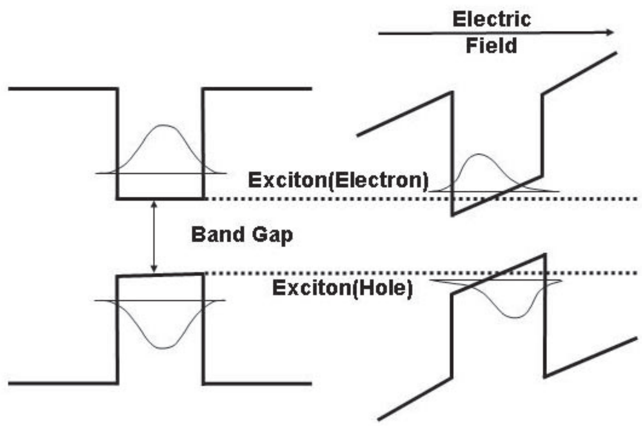


Figure IV.9. The potential well prevents the free movement of the electron and hole under the influence of an electric field.

Confined to a quantum well however, no dissociation of the exciton is possible, since the potential well acts as a barrier for movement of free electrons and holes, as shown in Figure IV.9. The application of bias shifts the electron and hole in opposite directions but they are prevented by the potential well from dissociating. However, the energy levels are altered due to the electric field which is also shown in the figure. The confinement of the exciton in the potential well also increases the binding energy by ~4 times the bulk value for a well thickness of 10 nm, less than the exciton size in three dimensions, ~30 nm. This increases the life time of the exciton (prevents its decay).

The practical quantum well optical resonator is operated as a reverse biased p-i-n diode. This structure includes an intrinsic (undoped) semiconductor layer between the p and n layers. The reverse bias ensures that the power dissipation is low. The schematic is shown in Figure IV.8. The quantum wells are located between the p and n regions and multiple wells are employed to increase the probability of finding an exciton. The wavelength of light is chosen to match the difference between the energy levels of the exciton at zero bias. Thus the device under bias is not transparent since the photons are absorbed. When an electric field is applied, as described above, the exciton energy levels shift and the photons in the incident light beam are no longer absorbed. Thus an electric field

operates an optical switch for high speed communication devices. Quantum behavior of a non-elementary quasi-particle excitation is "real" to the extent of being exploited in a technologically relevant device.

Laser

Light Amplification by Stimulated Emission of Radiation (LASER) is based on Einstein's realization that the emission of a photon by the transition from the high energy to low energy quantum state can also be "stimulated" by the presence of a second photon and that these stimulated photons will all be "coherent" (They will all be in a single quantum state). The theoretical understanding of stimulated emission, by Einstein, predates the development of the actual devices by close to forty years. The first devices actually operated in the microwave region and were called MASER (Microwave rather than Light). The laser is next to the transistor the most common quantum device in the market. While a transistor as a stand alone device is not very popular and useful, a laser has become a common accessory as a pointer, a precision surgical implement in medicine and for large scale decorative and visual display applications. It also has created more impact in the human mind as reflected in cartoons and science fiction.

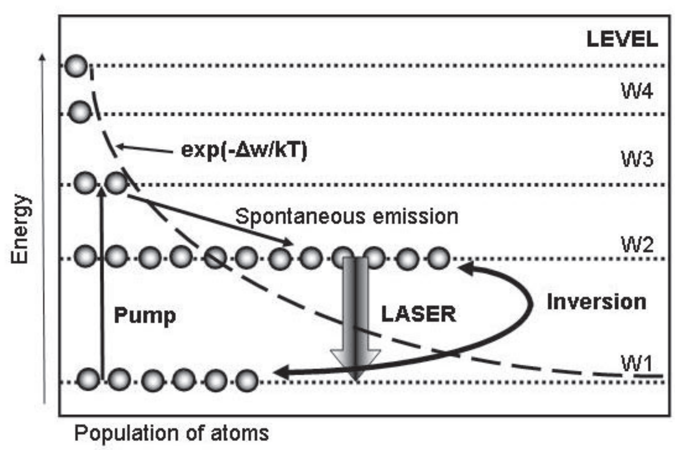


Figure IV.10 The population of atoms in various energy states during population inversion

Stimulated emission has two requirements, presence of a number of excited atoms, capable of emitting photons and means of stimulating them into radiating as a coherent quantum wave rather than as a normal thermal source. The number of atoms in a higher energy state is normally lower than that in the lower energy state as shown in Figure IV.10. Laser action requires "population inversion" (more atoms in the higher energy states). This is practically achieved by using a second source of radiation which is called the pump. The pump is used to excite the atoms to a state higher than the state from which stimulated emission is expected to take place. In Figure IV.10, the pump delivers the atoms in the lowest W1 state into the W3 state. These then decay rapidly into the W2 state by de-excitation. The material is chosen to have a rapid decay from W3 to W2 but a slower decay from W2 to W1. Pumping then causes a population inversion between W1 and W2 which permits laser action.

Figure IV.11 shows schematically the steps leading to laser action. The atoms in the ground state W1 are excited into W3 by pumping which is usually accomplished by optical means though electrical or even chemical pumping is possible. These then de-excite into W2 which then provide the initial stimulated photons shown schematically as waves. The atoms in

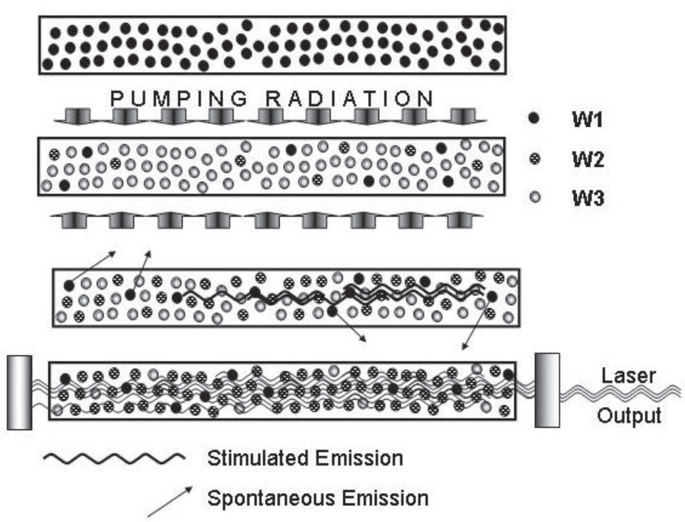


Figure IV.11 Schematic representation of laser action.

W2 can also decay spontaneously emitting photons in random directions shown schematically as arrows. The mirrors are used to reflect the initial coherent photons so that they in turn stimulate more excited atoms into stimulated emission. The number of photons in the coherent state continuously increases and a fraction is extracted from one of the two mirrors which is made partially reflecting for this purpose. Use of highly reflecting and parallel mirrors is necessary in practice to increase stimulated emission. All stimulated photons are in phase and have to be emitted in the same direction. This is the reason for the high directivity and exceptional frequency stability of laser and maser sources. Typically, the frequency stability of a maser (or monochromatic nature of a laser) is better than that of any alternate source by three or more orders of magnitude. Similarly the coherence length of lasers is several orders of magnitude larger than ordinary sources of light.

Some of the other interesting properties of coherent collection of bosons have been mentioned earlier. Generally speaking, creating a population inversion is extremely costly in terms of energy. It is only the exotic properties of laser radiation that make it a useful technological option. The one most important property of laser radiation that is immediately obvious is the directionality which is the key in applications ranging from surgery, machining, guided missiles and nuclear fusion to lighting effects in celebrations and music performances. The energy is transferred in a given direction unlike normal energy which would dissipate in all directions resulting in the familiar reduction of energy density as the square root of the distance. For all normal energy sources, absence of the square root dependence is an indication of violation of the law of conservation of energy. (Of course, this is never observed).

Hologram

Strictly speaking, a hologram is not a “quantum device”. The theory of a hologram was developed before the practical development of a laser. Some holograms were also fabricated using ordinary light. However, the very short coherence times of other sources of radiation limit their viability

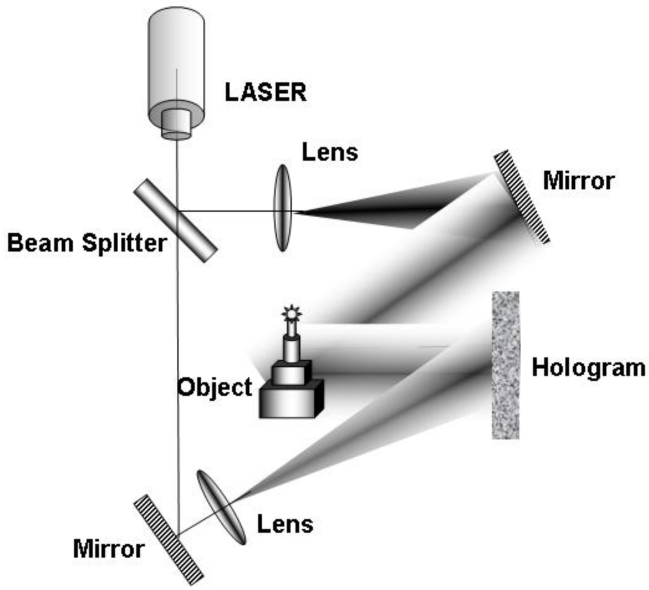


Figure IV.12 Schematic setup for preparing a hologram.

for preparing a hologram. The hologram has been included here because of the widespread application in art and advertisement usually as a three dimensional photograph. Stated briefly, a hologram is the interference pattern between a source and the light reflected by an object. Figure IV.12 shows the principle of preparing a hologram. A laser beam is split into two parts. One is used to illuminate the object. The light reflected by the object falls on the photographic plate which is also exposed to the original light beam. For interference, as has been discussed earlier, the phase of the two paths must be well defined; which means that the coherence length of the light source has to be larger than the difference in path lengths of the two interfering beams. It is at once obvious that for any meaningful hologram, the coherence length has to be comparable to the distance of the source and size of the object. This, for any macroscopic object, demands a laser as the source. The interference pattern is recorded on a photographic plate but the hologram cannot be directly viewed like a photograph. It has to be viewed with a laser source and the object appears to be suspended in space behind the hologram. The first interesting aspect about a hologram is that three dimensional perspective is obtained. The pictures

Figure IV.13 The famous example of a hologram showing the three dimensional imaging experience. The three views are of the same hologram. (Used from http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html Carl R. Nave, Department of Physics and Astronomy Georgia State University ©C.R. Nave, 2006)

in Figure IV.13 illustrate the point. All the three views are obtained from a single hologram by altering the position from which it is being viewed. The relative positions of the pawn and the king in the chess pieces is obvious.

Since the hologram is only an interference pattern, any part of the hologram contains the information regarding the entire object. Since the light reflected from all visible parts of the three dimensional object interfere, reconstruction of a three dimensional image is possible. Using only a portion of the hologram for reconstruction is also possible. However, this results in loss of image quality. Since the hologram is an interference pattern, the size of the image can be altered by changing the wavelength of the light used for viewing. Many variations of the basic hologram including viewing with white light have been developed and extensively used for art and advertisement.

Spin valve GMR head

The spin valve GMR head is a practical device that is present in virtually all the computer hard disks that are currently in use. It is unique among current commercial electronic devices since it directly exploits the quantum mechanical nature of the spin of an electron. Electronic devices which use spin have been labeled as spintronic devices. The schematic of a practical GMR head is shown in Figure IV.14. This consists of a four thin film layers. The first is labeled as the anti-ferromagnetic exchange film.

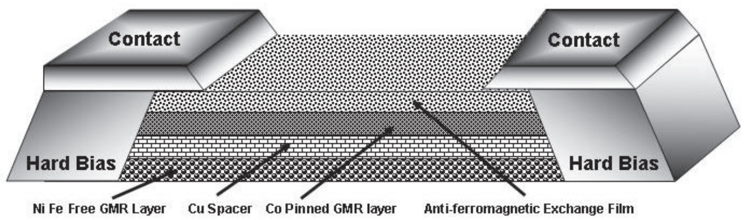


Figure IV.14 The schematic of a spin valve GMR head in computer hard disks.

The anti-ferromagnetism in this film pins the magnetization of the cobalt layer below it, that is, the magnetization of the cobalt layer is always fixed in a specific direction due to the presence of the first layer. This cobalt layer exhibits Giant Magneto Resistance (GMR). (The resistance change due to the application of a magnetic field is very large). This behaviour is essential for the operation of the device but the details are beyond the scope of this discussion. The third layer is non-magnetic copper which separates the Co-GMR layer with the fixed magnetization direction and the fourth layer which is a NiFe alloy also exhibiting GMR. The hard bias is a permanent magnet film used to lower noise. The magnetization direction of the NiFe layer is free. It can be either parallel or anti-parallel to the magnetization of the Co-GMR layer. The NiFe layer is very near the magnetic storage medium of the computer hard disk. This is typically a circular disk, which rotates while the head itself moves along the radius of the disk so that the head can reach any point on the disk. As is well known, the disk is separated into small regions each of which is a magnetic domain with a net magnetization. The direction of this magnetization for a bit “0” is opposite to that for bit “1”. For example, if each domain is considered to be a small magnet, the north pole can be close to the Ni-Fe layer when the data stored is “0” and the south pole when the data stored is “1”. During the operation of the head for reading the information on the disk, the free GMR layer comes close to the domain in the disk. The direction of magnetization will be aligned with the domain. It therefore depends on whether the information read in the recording medium is a “0” or “1”. The resistance of the device is low when the two directions (the magnetizations of the Co and NiFe GMR layers) are parallel and high

when they are anti-parallel. This difference in resistance is then amplified as a signal for reading the data stored on the disk. The differences in resistances arise because the scattering of the electrons, (which is the cause of resistance in all materials) depends on the direction of the spin of the electron. This practical device designed and operated based on the use of spin is a real challenge for discussing the philosophy of quantum theory in the last chapter.

Many other interesting demonstrations of spintronic devices have been proposed in research literature. As has been mentioned in the last chapter, the number of "spin up" and "spin down" electrons in a ferromagnetic material are not identical. Spin polarized electric current, that is, current consisting of electrons predominantly in one of the spin states can be produced by various means. For example, current extracted from a ferromagnet by tunneling through a barrier is spin polarized. Spin polarization has been demonstrated with materials like $La_{0.66}Sr_{0.34}MnO_3$ (LSMO) at low temperatures. Consider a domain or region of the ferromagnetic material containing predominantly electrons of a particular orientation of the spin. By injecting spin polarized electric current of the opposite spin orientation it is possible to reverse the direction the majority of the spins. This reverses the direction of magnetization in that domain or region of the material. Laser diodes are well established. The familiar laser pointer is an example. In this, laser radiation is emitted by injecting electric current into the semiconducting p-n junction. Obviously, pumping and mirrors etc., are different from the example discussed earlier. When spin polarized electric current is injected into p-n junctions fabricated using magnetic semiconductors, emission of polarized laser light has been observed.

Superconducting Quantum Interference Device (SQUID)

SQUID, as it is abbreviated, is extensively used for high sensitivity measurement of magnetic fields in physics, in medicine for imaging brain action and in mineral prospecting. A superconductor exhibits zero resistance because of the formation of Cooper Pairs, each of which consists of two electrons and a phonon which together behave as a boson. The

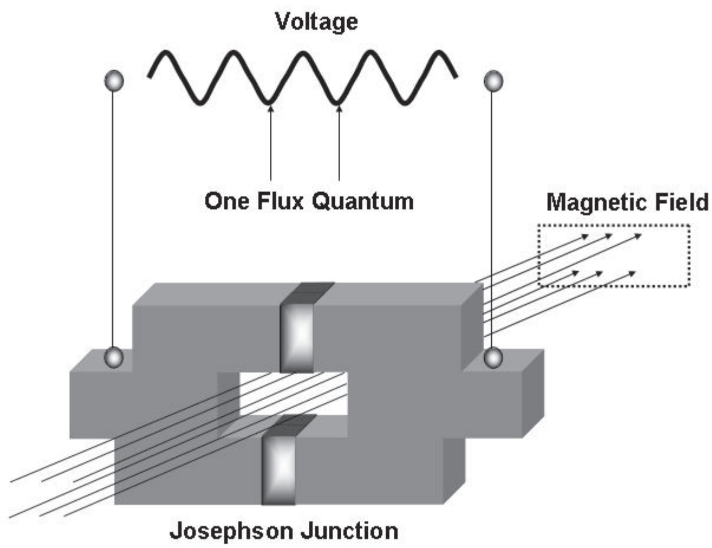


Figure IV.15 Schematic view of a SQUID consisting of a ring of superconducting material with two Josephson Junctions (Non-superconducting barriers).

superconductor is a perfect diamagnet; it expels magnetic field from its interior. Persistent currents are created, which flow in appropriate direction so as to create a magnetic field that is equal and opposite to the external field. Since the resistance is zero however, no energy is dissipated and the law of conservation of energy is not violated. Now, consider a ring of a superconductor which is subjected to a magnetic field. There will be a current flowing round the ring to cancel the magnetic field in the central cavity. Normally, there is a limit to the amount of super current that can be passed in a superconductor. When the external field is increased, at some stage the material cannot maintain the currents and the material looses the superconducting property and the diamagnetism.

Next consider, the ring shown in Figure IV.15 where the ring has two barriers incorporated in it. These are called Josephson Junctions and consist of a thin insulating layer separating the superconductors. As mentioned earlier, when the barrier is thin, the quantum particle has a probability of tunneling through the barrier. This is schematically shown in

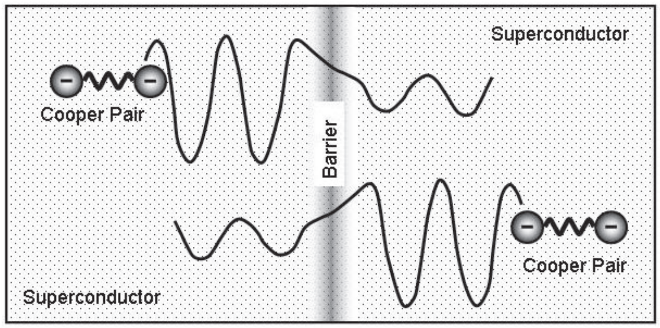


Figure IV.16 Tunneling of a Cooper pair through a barrier.

Figure IV.16. The Cooper Pair acts like a single quantum particle as it tunnels through the barrier. The amplitude and the phase of the wave are changed during the process. This is schematically shown in the figure. Since the amplitude changes rapidly inside the barrier, the oscillatory behaviour cannot be observed in that region. If the barrier were very thick, the amplitude would be too small for observing the Cooper Pair on the other side of the barrier.

A small amount of current can thus flow through the barrier due to tunneling of Cooper Pairs. The current through the barrier depends on the phase difference, which can have a maximum value of 2ð (as with all phase angles) and hence, as the phase difference increases, the magnitude of the current oscillates. At very low currents the phase difference is constant with time. In the SQUID, the currents in the two arms of the ring interfere, constructively, if the phase differences across the two barriers are equal. If the phase is opposite, the interference is destructive. The currents in the ring are changed when the magnetic field in the ring is altered. As the magnetic field increases, an oscillatory voltage is produced. As shown in Figure IV.15, a change in magnetic flux by one flux quantum or 2×10^{-15} weber causes the observed voltage to complete one cycle of variation. Obviously the SQUID is an extremely sensitive device. For comparison, the earth's magnetic field is quite weak but will still correspond to about 10^4 flux quanta/cm^2. This makes it an extremely accurate measuring instrument in the physics laboratory. Further it can be used for monitoring

small changes in magnetic field of the earth caused by mineral deposits. Till the advent of the functional NMR of the brain, electro-encephalogram of the brain using SQUIDS was the only means of detecting the changes occurring in the brain. These studies are extremely relevant not only for medical diagnostics but also in answering very important questions in neuroscience. When the current in the barrier increases, the phase difference across the barrier becomes proportional to time. Energy is lost as radiation of frequency 2eV/h (where e is the electron charge and h is Planck's constant), as the Cooper Pair crosses the Josephson Junction. In the reverse, exposure to radiation results in a dc voltage. This is now the accepted as the international standard of voltage. The SQUID is unique in that the word "quantum" appears in the name itself. Obviously it could not have been conceptualized without quantum understanding.

Carbon nanotube electronics

A carbon nanotube (CNT) is basically a few layers of graphite that have been rolled like a carpet and stitched up into a tube at the molecular level. It is an elongated version of the fullerene molecule C_{60}. Carbon nanotubes are now commercially available. Applications are being demonstrated in a variety of areas, ranging from high strength composites to chemical sensors. Carbon nanotube based electronic device is being considered as an example of a quantum device in the present chapter, even though it has not been commercialized. However, the electronic conduction in carbon nanotubes illustrates many quantum principles. Thus it is a very good example of designing a quantum device.

For electronic devices, single walled carbon nanotubes are used. This version of a CNT consists of a single layer of graphite with no dangling bonds along the surface and the ends being closed by half a fullerene molecule (The fullerene or C_{60} molecule consists of sixty carbon atoms and is shaped like a soccer ball). The CNT exhibits metallic or semiconducting nature depending primarily on its diameter. The band gap can be altered by adsorption of individual atoms or molecules (such as hydrogen and oxygen) on the CNT. Radial mechanical deformation also alters the electronic band structure. The molecular symmetry inherent in a

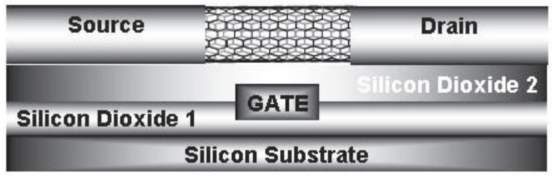


Figure IV.17 Schematic of a CNT based MOSFET. (Adapted from A. Javey, Q. Wang, A. Ural, Y. Li, and H. Dai, Carbon Nanotube Transistor Arrays for Multistage Complementary Logic and Ring Oscillators, Nano Letters 2, 930, 2002)

perfect carbon nanotube is lost due to bends or kinks which act as semiconducting junctions.

The second consequence of the one dimensional quantum nature of a carbon nanotube is that small angle scattering of electrons due to other electrons or phonons is absent. The only possible path for scattering an electron and causing resistance is along the tube in the backward direction. This has low probability and the conduction is ballistic or without scattering. Thus one notices a different quantum phenomenon. Unlike in a superconductor where scattering is prevented by Bose condensation, here one has limited scattering of fermions only due to dimensional reasons. Consequently, the resistance is not identically zero as for a superconductor.

In view of these attractions, development of carbon nanotube electronic devices is an important and active area of research. These devices provide low resistance and power dissipation. The major problems are technological such as organizing and assembling these nanometer size objects. Achieving this in an efficient way is beyond current technological capabilities. One example of a demonstration however is being presented.

Figure IV.17 shows the schematic of a carbon nanotube MOSFET. (The MOSFET is an established three terminal electronic device like the transistor. The current between the two outer electrodes called the source and drain is controlled by the electric voltage applied to the gate between them). In the CNT device, the gate is made of tungsten metal buried in between two layers of silicon dioxide while the source and

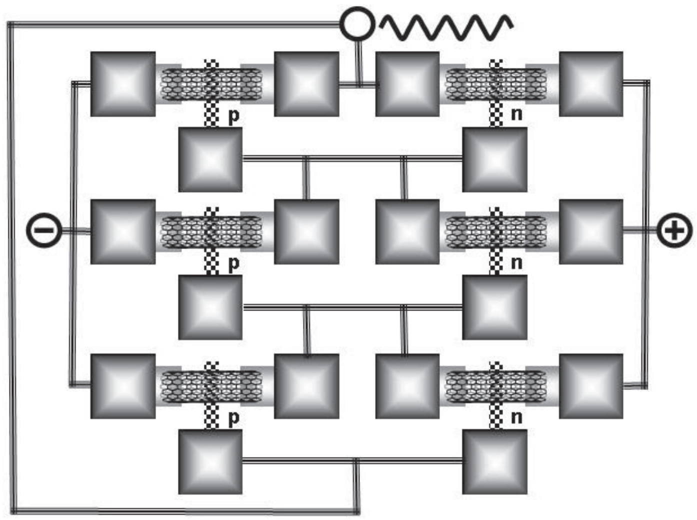


Figure IV.18 Schematic of the ring oscillator circuit formed using six CNT MOSFETs. (Adapted from A. Javey, Q. Wang, A. Ural, Y. Li, and H. Dai, Carbon Nanotube Transistor Arrays for Multistage Complementary Logic and Ring Oscillators, Nano Letters 2, 930, 2002)

drain are made of molybdenum. Forming metallic islands of this type needs only standard semiconductor processing technology and hence an assembly of large number of these contacts is easily fabricated. Molybdenum is chosen because carbon nanotubes grow on Mo metal, when exposed to carbon containing gases such as methane, at high temperatures. Carbon nanotubes would form between the source and the drain of at least some of the available contacts forming the source-drain connection. Six of these, three each of p and n type conduction are then selected for device operation and externally connected to form a demonstration circuit called a ring oscillator shown in Figure IV.18. The oscillator output has been demonstrated confirming the possibility of using one dimensional CNT transistors for advanced computers. Obviously, the process is quite cumbersome and not suitable for commercial use. A better way of organizing these tubes is required for commercial application. Self assembly may in future provide the capability for growing the tubes in the correct

places and organize electronic circuits from the bottom up. However, the demonstrations once again confirm quantum ideas in operation at low dimensions, an attribute these devices share with the HEMT.

Photonic crystal

A photonic crystal is a demonstration of extending quantum ideas to a new domain rather than a commercial technology. The area is being extensively investigated with the hope of realizing photonic devices analogous to electronic devices, in future. Order in the atomic arrangements, and consequently in the potential in which the electrons are confined, leads to forbidden energy bands in three dimensional materials. The ability to manipulate these energy bands is the basis for the development of electronic devices. The potential determines the velocity of electrons since it defines the energy. As is well known, refractive index is the ratio of the velocity of light in vacuum to that in the medium. Therefore there is a possibility of developing materials with photonic band gaps by creating materials with periodic change in the refractive index.

The consequences of periodic changes in refractive index have been investigated in the past. The colours exhibited by opal, the wings of butterflies and the feathers of the peacock are examples from natural world of local changes in refractive index, a phenomenon called “iridescence”. However, in the past, the dimensions over which the refractive index could be periodically changed, by assembling a collection of macroscopic objects in the laboratory, was very large. Photonic band gaps are realized only when the periodic changes in refractive index can be established at dimensions comparable to the wavelength. These gaps are range of wavelengths of light which cannot be propagated in the composite medium.

Recent technological developments have enabled the preparation of perfect spheres of polymers such as polystyrene and insulators such as glass. These are now commercially available at dimensions as small as 100 nm. Colloids can be formed with these spheres. Rate of sedimentation or settling of the colloidal particles is very slow, when the sphere diameter

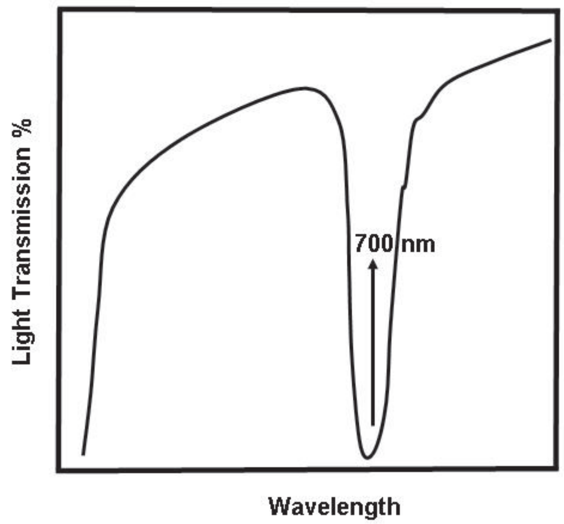


Figure IV.19. Sharp decrease in the transmission of a collection of spheres indicates the onset of a photonic bandgap. (Adapted from S.Kuaia, X. Hub, A. Hachlea and V. Truonga, High-quality colloidal photonic crystals obtained by optimizing growth parameters in a vertical deposition technique Journal of Crystal Growth 267, 317, 2004)

is reduced below 500 nm. As the colloids are dried, the spheres arrange into perfectly ordered close packing, due to surface tension in the evaporating film of the liquid. The order can be improved by using electric fields. Close packing of spheres of different sizes creates materials with different band gaps (colours). This is equivalent to altering the bandgap of a semiconductor by applying high pressure. The inter-atomic distance changes and consequently the bandgap also changes. Similarly when the sphere diameter is changed, the distance over which refractive index changes is altered. The refractive index of the sphere is different from that of the voids in between the spheres (usually air or a solvent). A direct measurement of the absorption in the solid as a function of wavelength exhibits a strong decrease at the specific wavelength, for example at 700 nm in Figure IV.19, which is the photonic bandgap. The demonstration of photonic crystals confirms the experimental utility of quantum ideas in a novel situation.

Quantum mechanical resonator.

The recent development, first of Micro Electro-Mechanical Systems (MEMS) and then their nano counterpart, NEMS, has resulted in the ability to fabricate very small mechanical components. These techniques use the fabrication processes used in the electronic industry for the fabrication of ultra small transistors. Fabrication of micron and submicron cantilevers, gears, motors etc. has been demonstrated. Mechanical parts with the resonance frequency of more then 1 GHz have also been experimentally realized and the dimensions are being reduced further. The question of whether such small mechanical artifacts, fabricated using lithographic techniques, exhibit quantum behaviour, just like molecules and atoms, is being experimentally explored. Many extremely interesting quantum experiments done recently will be discussed in the next chapter. However, the present example is coupled with other quantum devices since the attempt is to reduce the size of a mechanical resonator till it becomes a quantum resonator. It is also an example of a device that has been conceptualized but not yet actually realized, demonstrating that the development of newer quantum devices is an ongoing and unending process.

A mechanical resonator, vibrating at a frequency ν can be expected to have energy in multiples of $h\nu$. This is simple application of the Planck's quantization of a mechanical oscillator very much in the spirit in which phonons were conceptualized. However, the resonator, in the quantum limit must exhibit zero point energy. The energy must obey the relation $E = h\nu(N + ½)$. Since the Heisenberg principle applies even at absolute zero of temperature, if the $½h\nu$ were not included, the resonator energy at absolute zero of temperature would be identically zero, which would violate the uncertainty relation ÄEÄt $\leq h/2\pi$. Correspondingly, the resonator must exhibit an uncertainty in the position corresponding to zero point energy. The temperature at which the number N becomes less than unity, (when the motion is purely due to zero point energy) can be calculated. Below this temperature the only vibration of the resonator is due to zero point energy. For example, the temperature when the thermal energy is

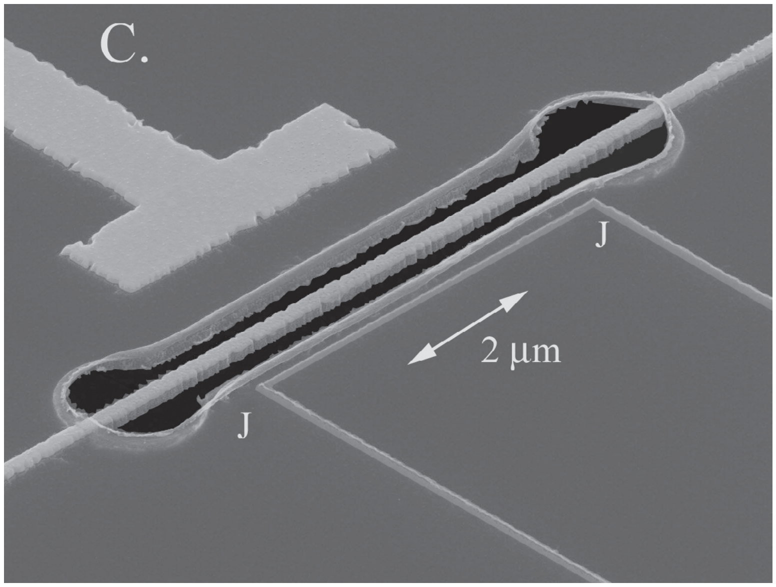


Figure IV.20 The nano resonator placed close to the gate of a transistor. The vibration results in change in capacitance and consequently a signal measuring the energy of the resonator. (Used with permission of K C Schwab)

less than the quantum energy of vibration, $kT < h\nu$ is ~ 50 mK for a 1 GHz resonator. Fabricating a mechanical resonator of resonant frequency 1 GHz and cooling it to below 50 mK are both relatively easy. Experimentally confirming the zero point motion of the resonator however is very difficult. The average distance over which the mechanical resonator moves depends directly on the energy available. The movement due to zero point energy, ($\frac{1}{2}h\nu$) is $(h/8m\pi^2\nu)^{1/2}$. For example, if the resonance frequency is 10 MHz and the mass is 10^{-15} kg, the above formula shows that this motion is only 10^{-14}m. Position detection schemes, capable of detecting such motion at these frequencies (Combination of the required bandwidth and sensitivity) have not yet been devised. Figure IV.20 shows the best that has been achieved so far. It consists of a 20 MHz nano resonator. The thin rod like structure which is placed close to the "T" shaped electrode connected electrically to the gate of a specially constructed transistor. As the resonator vibrates, the charge on the gate electrode, shown in the picture, changes

which results in an electronic signal that has been measured. In experiments performed at 60 mK, the researchers have achieved N values as low as ~58, nearly a factor of 60 from the quantum limit which corresponds to N <1 as mentioned above. Obviously the temperature has to be lowered significantly without loosing the electronic sensitivity. Quantum zero point motion has actually been confirmed when the vibrating resonator is replaced by a carbon nanotube but this is similar to the observation of zero point motion in a number of molecules and does not carry the significance of observing quantum behaviour of a small mechanical part machined according to human design.

Technological applications of quantum ideas demonstrated here with representative examples are quite impressive. Simple well established ideas of energy bands, quantum mechanical tunneling, other quantum objects such as the excitons, two and one dimensional systems and Bose condensation have all been employed for making viable devices which in many cases have been commercialized. This translates quantum ideas from theoretical physics to the real world.

V
Performing "Thought"Experments

The previous chapters provide a glimpse of the bewildering success of the quantum approach. Planck's law, introduced by Max Planck as an ad hoc assumption of the energy of a vibration of an electromagnetic mode in a cavity, appears to hold good for the energy of a quantum of lattice vibration, an oscillation of a charge in a metal, the rotation of a quantity of superfluid helium and so on. The quantum description is necessary to understand the experiments performed on electrons and photons. As seen subsequently, a description on the basis of other quantum objects became necessary for understanding collective behaviour. The technological utility is as unquestionable as the theoretical necessity for understanding various phenomena.

The mathematical structure of quantum theory has always been a source of great wonder and worry. This started with the first application of Planck's quantization approach to the hydrogen atom by Bohr. Einstein immediately noticed that a degree of randomness has been introduced since the theory does not actually describe the transition from one quantum level to another. The Heisenberg uncertainty relation, limiting the experimental ability to simultaneously determine the momentum and position of a particle has been another strange feature of the quantum description. Bohr subsequently developed the idea, that experiments can be performed to determine the particle properties (position) or wave properties (momentum) but not both simultaneously. Einstein pointed out that a natural consequence of the quantum description is entanglement. The properties of quantum systems separated by macroscopic distances are interrelated. Measurement on one leads to instantaneous consequence for the other.

The wave function of a quantum system evolves continuously and collapses during measurement. The separation between the quantum system and the measurement system is not well defined. The measurement system itself consists of quantum particles and in many experiments, the quantum particles appear to "know" the presence of equipment such as polarizers and cavities. The mechanism by which quantum weirdness disappears in macroscopic systems is also not clear. Thus "gadanken" or thought experiments were employed by Einstein, Heisenberg, Schrödinger and other scientists, to illustrate the above problems and probe the relation between nature and quantum theory.

The use of thought experiments has a wonderful history. Galileo used the description of an imaginary experiment to illustrate relative motion and invariance. Newton used the example of a rotating bucket full of water to illustrate absolute space. Maxwell imagined a demon that could violate the second law of thermodynamics. Einstein was a master in the use of the thought experiments. He illustrated the incompatibility between Newtonian physics and Maxwell's equations by imagining an experimenter who moves along the crest of a light wave. The observer traveling with that velocity would be unable to see light, determining the absolute rather than relative velocity. He used an imaginary elevator moving with uniform acceleration to illustrate the principle of equivalence, that acceleration and gravity are equivalent. As shall be seen in this chapter, he was confident of finding the limitation of quantum descriptions since he could visualize a thought experiment (The famous Einstein, Podolsky, Rosen or EPR paradox discussed below) to indicate the limitations of quantum theory.

Not very surprisingly, the thought experiments, visualized by the famous scientists such as Einstein and Heisenberg, to illustrate the counter-intuitive nature of quantum ideas, have triggered an effort to actually perform experiments illustrating these aspects of quantum mechanics. In the process, the qualitative and descriptive aspects have been converted into precise quantitative science. Some of these will be discussed in this chapter. These experiments can be seen as extremely precise justifications of the quantum mechanical methodology. At the same time, the utilization of these experiments in practical technology, very much in the spirit of the last

chapter on quantum designed devices has been an ongoing process. Since only few of these have become practical, the technological developments have been included in this chapter. The experimental support to the mathematical description of quantum mechanics is really mind boggling. One comes close to understanding Feynman when he dismisses the whole philosophy by stating that "no one really understands quantum mechanics". This will be briefly summarized in the next chapter.

Entanglement

Einstein developed the famous Einstein, Podolsky and Rosen (EPR) thought experiment to illustrate his reason for considering quantum explanation to be incomplete. This original paper considered the use of momentum and position as variables. The problem is now explained using the argument of David Bohm who converted the essence into entangled spins. The present experiments are performed using photons and the polarization (spin) is determined.

Imagine a system consisting of two entangled quantum particles moving in opposite directions. The total spin of the two particles is zero but the individual values are not defined. When an experiment is performed to verify the spin or polarization of one of the particles in a given direction (say the horizontal), the result would have two possibilities. Either the spin is in positive horizontal direction or in the negative horizontal direction. As soon as the experiment is performed, the spin of the second particle will be observed to be opposite to that of the first. But the fact that an experiment is being performed in the horizontal direction is not conveyed to the second particle. It could have been performed in the horizontal, vertical or in any other random direction. In each case the measurement would provide either a positive or negative result. Instantaneously, the second particle will be in the spin state opposite to the measured value of the spin of the first particle. Einstein correctly pointed out that quantum mechanics does not limit the physical size of a wave function. Thus, the measurement does not violate quantum description.

At this point, it is necessary to introduce the concept of the "collapse" of a wave function. Once the wave function is defined it continues to evolve or changes continuously over all the possible experimentally observable results. When the measurement is performed, only one of the possibilities is realized. At this point, the wave function is said to collapse. Let us consider the single electron interference experiment introduced earlier. When an electron leaves the source, it has a finite probability for arriving at any point of the screen. Thus the wave function should reflect all these possibilities. The probability can be large for example in the regions where eventually bright fringes form or small in the region where dark fringes form. However all the possibilities should emerge from the wave function. Once the electron has arrived on the screen, one of the possibilities has been actually realized. At this point the wave function is said to have collapsed providing the probability for the electron to arrive at this point on the screen, which can be large or small as the case maybe. Now when two entangled particles are considered, the wave function should provide information about both the particles. The wave function of the two particles "collapses simultaneously". Then the polarization or spin of one of the particles can be probed in any direction. This is equivalent to experimentally determining if the electron has arrived at any given point on the screen. The simultaneous collapse predicts that the spins of the two particles will be experimentally observed to be opposed to one another.

Bell converted the qualitative thought experiment into a mathematical inequality, which distinguishes between quantum description and alternate theories which invoke "hidden variables" which propagate information in a classical way. Einstein, in arguing that quantum description was incomplete, assumed the existence of a classical mechanism which would transmit the information about the orientation of the first particle to travel to the second. Violation of the Bell's inequality therefore is an experimental confirmation that the collapse of the wave function, resulting in the second particle, exhibiting the spin value, opposed to the experimentally determined value of the first particle takes place without classical communication (at velocities smaller than the velocity of light) between the two particles. Current experiments probe the validity of the quantum description by confirming violation of Bell's inequality.

Such experiments have become possible due to the development of a process known as "parametric down conversion". This to date is the only easy means of creating entangled quantum objects. Normally, when light of a particular wavelength is incident on a material, it is absorbed and re-radiated at the same wavelength. In solids, energy transfer between the phonons in the solid and the incident radiation results in Raman scattering. This results in scattered radiation at slightly different wavelengths. Compton scattering is another possibility where the photon momentum is transferred and consequently a change in wavelength is observed. These were briefly described earlier but the change in wavelength is quite small.

In a nonlinear material on the other hand, the output consists of two photons of wavelength 2ë replacing one photon of wavelength ë. This is however a very inefficient process and only a very small fraction of the incident photons are converted. The key point is that the polarizations of the two photons generated will be anti-correlated; the polarization of one photon is opposite to that of the other. If the polarization of one of the photons makes an angle î, the second will have a polarization along î + ð/2. This is called parametric down conversion since the energy of the photon is reduced. If the wave picture is employed, the "nonlinearity" of the response of the material becomes obvious. If the output for small values of the amplitude of the wave is amplified by a factor "m" for example, the amplification for large values of the amplitude is less than "m". This is an example of nonlinearity. When the output is nonlinear, the output wave is distorted. Fourier showed that such a distorted wave can be considered to be the sum of the undistorted wave and waves with multiples of the

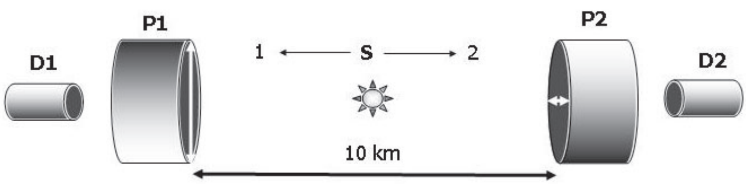


Figure V.1 Schematic of the EPR experiment which is performed using entangled photons traveling more than 10 km.

wavelength. The component with the wavelength twice the original value is the result of nonlinearity.

The typical experimental setup for performing the EPR experiment in practice is schematically shown in Figure V.1. It consists of a source S (a crystal exhibiting nonlinear optical response such as Bismuth Gallium Oxide (BGO)) of down converted (from ë to 2ë) photons 1 and 2 which travel in opposite directions. This replicates the essential idea of the EPR thought experiment of creating two entangled quantum objects (the original paper actually considers particles and not photons). Polarizers, P1 and P2 are placed in both paths. The output from individual detectors D1 and D2 does not depend on the orientation of the individual polarizers. For example, the output from D1 does not depend on the direction of the polarizer P1. This is understandable since there is no reason for the photons to have any specific direction of polarization. In the experiment, the directions of the orientation of the polarizers P1 and P2 can be individually varied arbitrarily. The data are examined to identify the conditions under which the outputs of the two detectors are correlated; photons are detected by both D1 and D2 at the same time. These photons correspond to one pair of photons created from a single down conversion. The correlation is observed when the difference between the two angles of polarization is exactly ð/2. Since the orientations are set at random, the mere fact of measuring the polarization of one photon along a direction î ensures that the other is in state î + ð/2.

The distance between the two polarizers is large. A series of experiments were performed with increasing accuracy and distance. Distances larger than 10 km have been experimentally investigated. Theoretically the correlations can last as far as infinity. If the measurement angle î is communicated from one polarizer to the other, the communication must take a finite time equal to the distance between the two divided by the velocity of light. The experiments confirm that the correlation can be detected even before the elapse of this time. The simplest description of the experiment would be to assume that the wave function of the two photons should be considered to evolve with time, irrespective of the

distance between them and that it collapses at the time of measurement. A minority (mostly non-physicists) however continue to question if the experiments are completely conclusive. These experiments suggest that introducing a hidden variable into the theory will not be sufficient to avoid the quantum mysteries.

As an example of hidden variables one can consider an ideal gas consisting of individual particles, each traveling with a velocity which cannot be directly determined or observed. However, the momentum transferred by colliding with the walls of the container is an observable variable- namely the pressure. In the ideal gas, the pressure depends on the probabilities for the particles to have various velocities. The velocities of all particles are not determined but they exist irrespective of whether or not they are measured. Quantum variables such as "spin" in contrast do not have precise values until measured. Similarly, it is possible to imagine the existence of variables underlying the position and momentum of an electron which do not have strange properties. No hidden variable theory with out "counter-intuitive" features can explain these experiments.

The origin of complementarity

Loosely speaking complementarity is the name given to the experimental observations described in chapter II. A quantum particle like

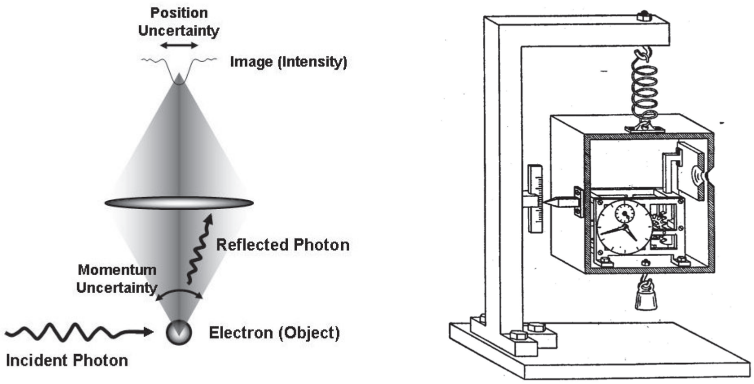


Figure V.2 The Heisenberg microscope and the Einstein photon weighing apparatus.

the photon seems to behave like a particle in some experiments and like a wave in others. Simple approaches seem to describe the electrons and photons as classical particles and also as waves, with the Heisenberg uncertainty relation linking these descriptions. The Heisenberg uncertainty principle is usually introduced using a gamma ray microscope as a thought experiment.

This is a simple microscope that is represented in Figure V.2 as a simple lens. The wavelength of light used for illumination in the microscope is decreased to increase the accuracy with which the position of an electron can be defined. This is the reason that an electron microscope gives much higher magnification than an optical microscope. The de Broglie wavelength of electrons is a thousand times smaller than the wavelength of visible light. As the wavelength of the light decreases its momentum increases, since the photon momentum as defined in the Compton effect experiment is photon energy divided by the velocity of light (hí/c). Thus a photon in the Heisenberg microscope collides with the object under observation very much like any collision of billiard balls. Momentum is transferred from the photon to the electron which is being observed. The magnitude and direction of the momentum transfer cannot be precisely known. Even in the simple lens, the photon may have entered along any of the paths enclosed by the cone of acceptance shown shaded in the Figure V.2. As the wavelength is decreased to increase accuracy with which the position is known, the momentum of the electron after observation becomes less accurately defined. Thus the error in the determination of the momentum of the electron increases as the error in the determination of its position decreases.

Einstein tried to counter this "limitation in principle" and came up with a series of ingenious thought experiments wherein the position and momentum of the quantum object (or equivalently the energy and time) could in principle be simultaneously measured with infinite accuracy. The weight box experiment also shown in Figure V.2 is the most famous. Einstein imagined a box with a shutter that can be opened and closed at a given instance to permit one photon to escape. The box itself is suspended on a spring balance to determine the loss in mass or energy. Thus Einstein

argued that both the time and energy can be determined to arbitrary accuracy. Bohr used theory of relativity (ironically Einstein's own theory making this experiment very popular) to argue that when the weight changes, the clock used to measure the time changes its position in the gravitational field and this changes the accuracy of the time measurement. This shows that the weight box experiment cannot simultaneously measure both energy and time with infinite accuracy. Bohr subsequently developed complementarity as a corner stone of his interpretation of quantum mechanics, with the idea that experiments "can" be performed to determine the particle properties (position, time) or wave properties (momentum, energy) but not both simultaneously.

Is it possible that the particle behaves both as a particle and a wave in the same experiment? More importantly, can the wave nature be attributed to the transfer of momentum from the particle as in the Heisenberg microscope? These quasi-philosophical questions have recently become amenable to experimental investigation. The first experiment is schematically shown in Figure V.3. Two photons are created by parametric down conversion (as in the EPR experiment mentioned earlier) from a single photon of 351.1 nm Ar ion laser light. One of the photons is collected in Detector 1 to check that the photons being detected in Detector 2 and Detector 3 are not photons due to stray external light. Only those detected

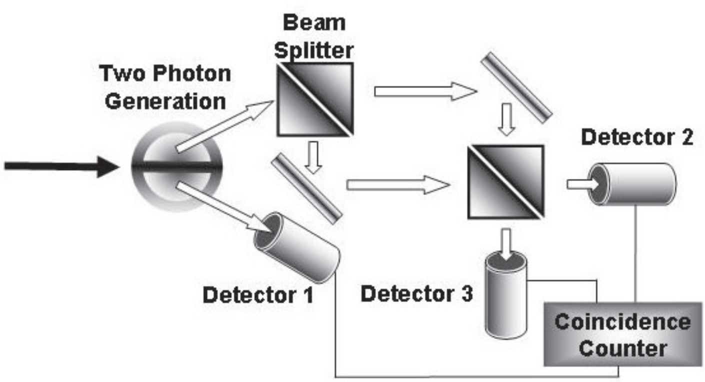


Figure V.3 Schematic of the experimental setup where the photons exhibit particle and wave nature simultaneously.

in D2 and D3 correlated with D1 are selected for further experiment. This improves the accuracy by eliminating the influence of stray light. The light, before it reaches the two detectors is passed through an interferometer. The light is split into two parts in the first beam splitter and recombined in the second. If the intensity of light is sufficiently large, the beam splitter ensures that 50% of the intensity is observed in each of the two paths. The output after the second beam splitter (acting in reverse, to combine the two beams) is equal to the original input (100%). In a modern system, the losses in such devices can be made very small and are ignored for the discussion. Using the coincidence counter, the arrival of a second of the pair (One is detected in D1) at D2 and D3 is monitored. The second photon never arrives at both D2 and D3 simultaneously. Either it appears at D2 (D1 and D2 are correlated) or at D3 (D1 and D3 are observed to be correlated). Since the second photon from the pair is not simultaneously observed at both D1 and D2, the best description of the experiment is the photon "behaves like a particle".

However the light in the interferometer is divided into two parts using a special type of beam splitter shown schematically. This consists of two prisms which are placed close to one another, but not actually touching, with the gap being comparable to the wavelength of light employed. When light is considered as a wave, the operation of the device is understandable. The wave tunnels into the air gap and 50% of the wave travels in one direction and 50% in the other. In the second device these two parts are combined again. Thus the proper description of the light in the beam splitter is a wave description. Contrary to the expectation of Bohr, experiments can be designed which need both particle and wave descriptions to be employed simultaneously. It should be noted that the experiment contradicts only the descriptive account of Bohr, not the mathematics of quantum theory. The results are in complete agreement with quantum theory.

If the beam splitter is replaced with partially transmitting mirror, 50% of the light is reflected and the rest transmitted. In this case the light in the entire experiment is described as particles. A calcite single crystal can also be used as the beam splitter. Calcite is a birefringent material. It separates an incident ray of light into two parts called the ordinary and the

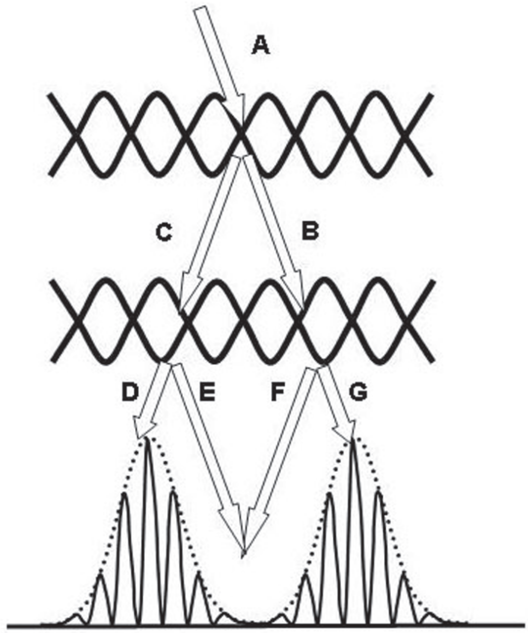


Figure V.4. Schematic view of rubidium atoms being diffracted by two standing waves. of laser light. The interference pattern is only observed when the path of the atoms is not detected. (Reprinted with permission from S. Duerr, T. Nonn and G. Rempe , Origin of quantum-mechanical complimentarity probed by a 'which-way' experiment in an atom interferometer, Nature 395, 33, 1998, © Macmillan Publishers Ltd. 1998)

extraordinary rays. Using the calcite crystal, it once again becomes necessary to use both particle and wave descriptions of the photon in the same experiment.

Another experiment has shown that the uncertainty is not caused by the transfer of momentum from the photon to the particle as described in the microscope thought experiment. The schematic of this experiment is shown in Figure V.4. The experiment is carried out using a beam of Rb atoms (laser cooled using the process described earlier, while discussing Bose condensation). These atoms are diffracted using two standing waves created by laser light as shown in picture. The beam A is divided into

beams B and C. They are further subdivided at the second optical grating into D, E, F and G. D and E (Also F and G) are aligned to permit interference. The interference pattern is shown. Using microwaves, the path information, the distinction between beams D and E (equivalently F and G) is experimentally determined. (These details are not shown.) As soon as this information is collected, that is, the path taken by an atom as it moves from the beam A to the detector is known, the interference pattern is lost. This is shown in Figure V.4 as the envelope of the diffraction pattern. The difference from a conventional two slit experiment is the use of microwaves to probe the Rb atoms. The momentum transferred by microwave photons, (which have a very large wavelength and hence very small energy) to the relatively heavy Rb atoms is too small. This cannot have caused the disturbance in the positions of the Rb atoms leading to the loss of the interference pattern.

Single electron and single photon interference experiments were discussed in Chapter II as evidence of the electron and photon being neither a particle nor a wave. Individual photons and electrons arrive at a screen as particles while the interference pattern of the waves builds up when sufficient number of particles are collected on the screen. This was originally a thought experiment suggested by Feynman, who also discussed a double slit experiment, with unreliable detectors at the two slits which monitor the movement of the photon through the slit. As is well known, if the detectors are used to know the path of the photon, the interference patterns disappear. The new variation here is the use of unreliable detectors. The intensity of the interference patterns in such an experiment is expected to decrease as the reliability of the detectors used increases. Such controlled experiments have not yet been performed. It is obvious that the experiment described is an example of absolutely reliable detectors and hence determining the path of the quantum particles causes the interference pattern to disappear.

Quantum erasers

We now consider quantum eraser experiments where, the information regarding the path followed by the particles is erased to recover

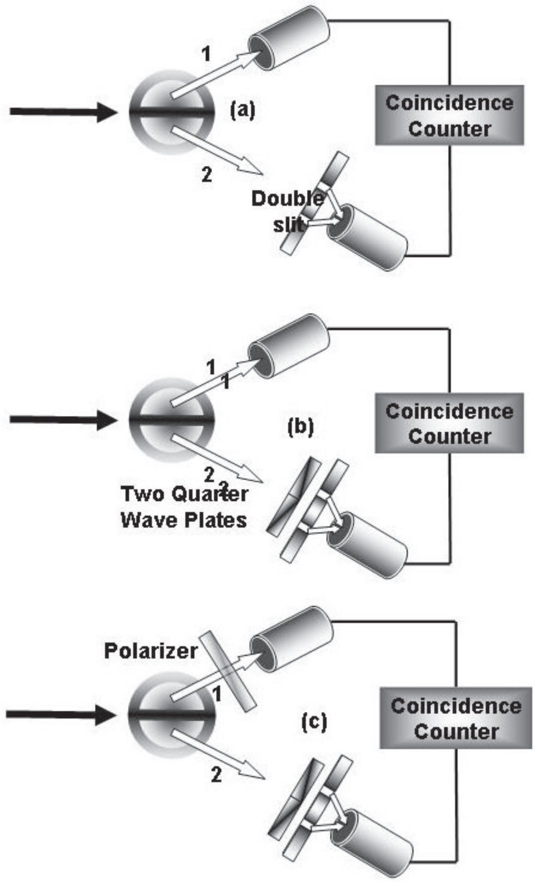


Figure V.5 Schematic views of an experiment demonstration quantum erasing. Interference if observed in both (a) and (c) but not in (b) . Results are shown in Figure V.6.

the interference. The schematic of the experiment is shown in Figure V.5. As in the experiments described above, parametric down conversion is used to create two correlated photons, 1 and 2. Figure V.5(a) shows photon 2 passing through a double slit before it reaches a detector which is moved normal to the slits. Photon 1 is incident on a detector. As in the previous experiments, only coincidences between the two detectors, in the coincidence counter, are used as reliable data. This experiment is equivalent to the familiar double slit experiment and as expected, interference

fringes are seen in the results presented schematically in Figure V.6 (Curve (a)). Next, two quarter wave plates are placed in front of the double slit (Figure V.5(b)). A quarter wave plate is a device that splits the incident light into two parts and adds a phase difference before they are recombined. The splitting is possible, in birefringent materials, such as calcite, which separate an incident ray of light into two parts, called the ordinary and the extraordinary rays. The velocities of the ordinary and extraordinary light rays in such a medium are different. When they both emerge from an equal thickness of the material, there is a phase difference between them. The quarter wave plate is suitably made so that the phase difference converts the light into circularly polarized light. In the language of picket fences introduced in chapter II, the direction of the polarization of a circularly polarized light continuously rotates. Obviously the rotation can be either clockwise or anti-clockwise. The quarter wave plate can either produce "right" or "left" circularly polarized light.

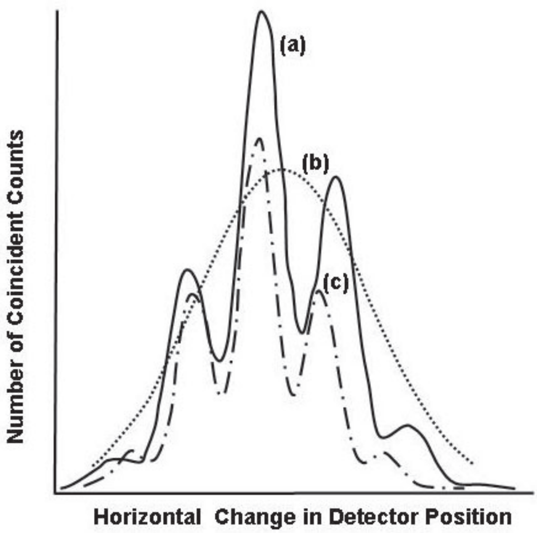


Figure V.6 Results obtained for the experiments schematically shown in Figure V.5. Interference is observed in (a) and (c). (Adapted from S. P. Walborn, M. O. Terra Cunha, S. Padua, and C. H. Monken, Double-slit quantum eraser Physical Review A , 65, 33818, 2002)

In the experiment, shown in Figure V.5(b), if the photon is passing through one slit, it is right circularly polarized and if the photon is passing through the other slit, it is left circularly polarized. If the polarization of photon 1 is known, the polarization of photon 2 is also known since they have opposite polarizations (The pair was produced by down conversion). Then the polarization of the photon detected after passing through the double slit, will reveal whether the photon has passed though the "right" or "left" quarter wave plates. This polarization measurement will therefore reveal the slit through which it has passed. Interference pattern is not expected, once the quarter wave plates are introduced. The results shown in Figure V.6 (curve(b)) confirm that the interference pattern is lost.

The polarization of photon 2, after it has passed through the quarter wave plates and the double slit is not sufficient to identify the slit through which the photon has passed. The polarization of the photon 2, incident on the quarter wave plate is necessary. The two quarter wave plates merely produce opposite sense of rotation for the incident photon. If the polarization of photon 1 was not known, the polarization of photon 2, incident on the quarter wave plate is also not known. In the schematic shown in Figure V.5(c), a polarizer with unknown direction of polarization is placed in the path of photon 1. Thus its polarization information is lost. It is no longer possible to identify the polarization of photon 2, incident on the slits. Thus, it is not possible to identify the slit through which the photon has passed, by determining its polarization. As predicted quantum mechanically, the interference pattern is restored as shown Figure V.6 (curve (c)). This is called a quantum eraser experiment. The information available in the system, the polarization of the two photons, is erased by introducing the polarizer without disturbing the photon incident on the double slit. This erasure restores the interference phenomenon even though the path of the photons undergoing interference is not disturbed.

Significantly, the restoration of the interference is observed even when the path of the photon 1 is increased so that the photon 2 is detected before photon 1 encounters the polarizer. This leaves the quantum mystery deeper. How can presence of the polarizer in the path of photon 1 influence

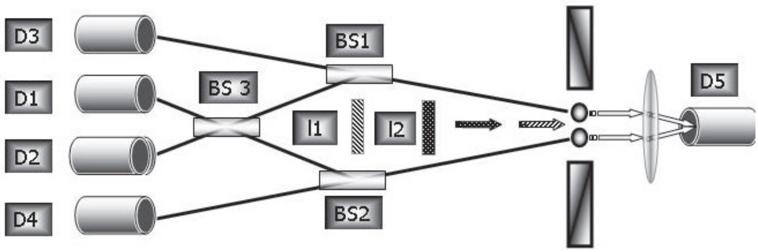


Figure V.7 Schematic of a quantum eraser experiment involving two atoms trapped using laser cooling.

photon 2, without any interaction between the two? The results are once again in perfect agreement with quantum predictions but describing the experiment is impossible other than making a general statement that the wave function of the correlated photons is as large as the experimental setup!

A second quantum eraser experiment is shown schematically in Figure V.7. In this, the two slits are replaced by two laser cooled atoms, localized using magnetic traps. The light pulse ℓ_1 is tuned to excite the atoms into state 4 from state 1 as shown in Figure V.8. The atoms decay into state 2 emitting radiation which is detected by the detector D5. The second light pulse, ℓ_2 is tuned to excite the atoms into state 3 from state 2 after which they decay back to state 1 emitting photons which are detected by the four detectors, D1–D4. Three beam splitters, BS1-3 are employed as shown. Obviously a photon detected by D3 can only be emitted by the

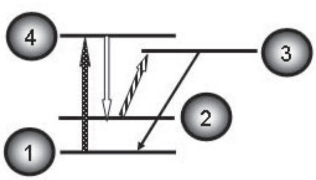


Figure V.8 Excitation states of the cold atoms. The patterns in the arrows match with those used in figure V.7.

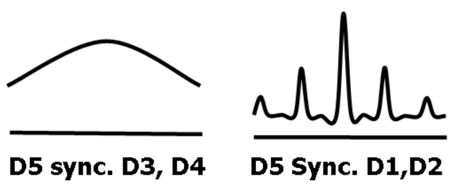


Figure V.9 Interference pattern is present in the output of the moving detector D5 only if the photons are correlated with D1 and D2.

upper atom while the photon detected by D4 can only be emitted by the lower atom. Only the paths "atom ➔ BS1 ➔ D3" and "atom ➔ BS2 ➔ D4" are possible. Photons detected by D1 and D2 cannot be unambiguously ascribed to either of the two atoms. The paths "atom ➔ BS1 ➔ BS3 ➔ D1" and "atom ➔ BS2 ➔ BS3 ➔ D1" are possible. Photons from both atoms are detected by D5. Thus, in principle, interference is possible, since a photon detected by D5 cannot be unambiguously attributed, to either the upper or the lower atoms. When D5 is moved, usual interference pattern is observed. When the photon detection in D1–D4 is correlated with D5, the results are striking. The photon interference is present when the information regarding the possible source is unknown (D1and D2). The interference is absent when the information is known (D3 and D4). Results are schematically shown in Figure V.9. The results are not surprising in light of quantum mechanical theory. However, the results are intriguing, from the perspective of the nature of reality all this represents as shall be discussed in the final chapter.

The Schrödinger's cat

The Schrödinger's cat is perhaps the most famous of all quantum thought experiments, as it appeals to most, as the key essence of quantum theories. Basically the question asked is very simple. Why is "quantum superposition" not observed in "macroscopic objects"? The quantum theoretical description includes "superposition" as an important attribute and quantum superposition of quantum objects has to be assumed for predicting many experimental results. The simplest example is the phenomenon of radioactivity. In the case of a single radioactive atom, the actual state of the nucleus is a superposition of the decayed and un-decayed states of the nucleus. This assumption alone permits a theoretical prediction of the law of radioactive decay. The number of radioactive atoms which decay in a fixed interval of time is dependent on the total number of such atoms present in the sample. For example, if one starts with one gram of radioactive iodine I_{131} whose half life is 8.07 days, 0.5 grams will decay in the first 8.07 days. However even after 1500 years, all radioactive atoms have not decayed, ~10 ìg of radioactive atoms are still present. The time after which a single radioactive atom decays is completely indeterminate.

This type of randomness was encountered earlier. The location on the screen at which a particular electron would be observed in the single electron interference experiment is indeterminate. Only the probability can be known and calculated using quantum theory. The quantum object, in this case the iodine atom can be in two states. It can either be in the excited state, from which it would decay or it could be in the low energy decayed state. Till the actual measurement is completed, the atom is in neither state but is in the superposition of both states. Earlier, describing the spin or polarization, it was demonstrated that the spin has no "value till it is actually measured". This "not having any value till it is measured" is an essential character of quantum theory. Experiments contradict the possibility that the spin has a value before measurement. The spin of a particle is a superposition of all the possibilities and collapses into the final value after measurement. The difference with the radioactive atom is that there are only two possibilities while for the direction of spin there are infinite possibilities.

Having accepted that superposition of states is experimentally confirmed for microscopic objects, Schrödinger considers a macroscopic body which has two possible states; in this case, a cat which is either dead or alive. He considered a box inside which a cat is placed along with a vial of poisonous gas and a mechanism which releases the gas after the decay of a single radioactive nucleus. Everyone agrees that till the nucleus decays, the cat is alive, afterwards it is dead. When the nucleus is considered as a superposition of the decayed and excited states, why cannot the cat be in a state of superposition of being "dead" and "alive"? Experimentally, the cat is never a superposition of a "dead cat" and a "live cat". The question that this thought experiment raises is fundamental. Why do macroscopic bodies not show superposition of states?

One easy answer emerges from the de Broglie wavelength. Since this decreases with increasing mass, the wavelength for any macroscopic body is too small to permit superposition and other quantum attributes. It should be recognized that merely increasing the number of particles does not necessarily convert a quantum object which exhibits superposition to a non-quantum object that does not. A laser beam is a good example. The

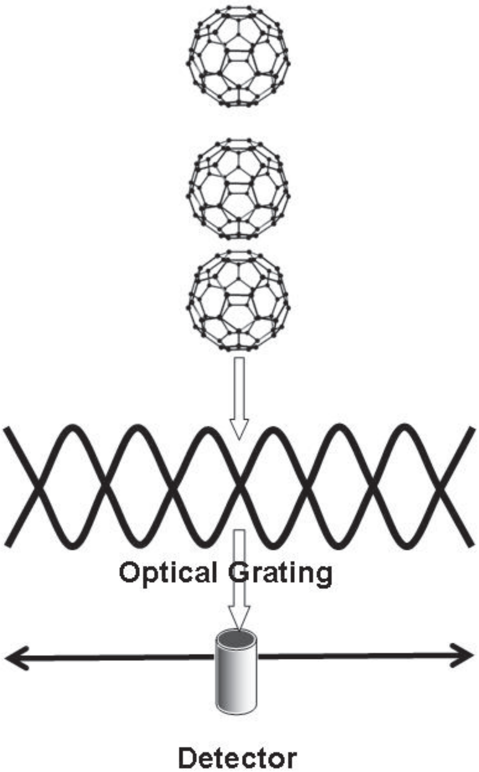


Figure V.10 Schematic view of fullerene molecules diffracted by an optical grating.

number of photons in a laser beam is very large; ~10^{15} for a 1 mW He-Ne laser. It does exhibit a coherent quantum state. Increasing the size is also not sufficient. The correlated photons in the EPR experiment described above show quantum correlation at distances as large as several kilometers. Thus merely the size or the number of particles in itself is not a criterion for transition to the non-superposition regime. In recent years, a series of experiments have been performed, increasing the complexity of the bodies which demonstrate quantum behaviour.

The largest clusters of individual atoms that have exhibited quantum nature are the fullerene molecules and carbon nanotubes. It was mentioned, in the discussion on quantum mechanical resonators, that the zero point motion of a carbon nanotube was experimentally demonstrated. More

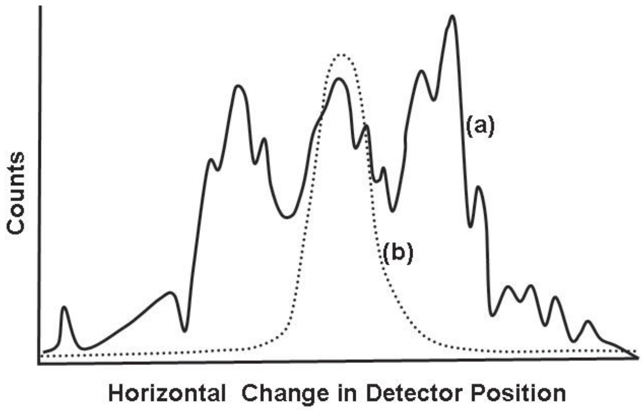


Figure V.11 The diffraction pattern obtained due to wave nature of fullerene molecules (b) is compared with the absence (a) when the optical grating is absent. (Adapted from O. Nairz, B. Brezger, M. Arndt, and A. Zeilinger, Diffraction of Complex Molecules by Structures Made of Light, Physical Review Letters, 87, 160401, 2001)

specific evidence is the observation of two slit interference patterns by C_{60} and C_{70} molecules. The experiment is schematically shown in Figure V.10. The beam of fullerene molecules from an oven is incident on a standing wave pattern created by laser light and the detector movement shows characteristic interference fringes shown in Figure V.11. The

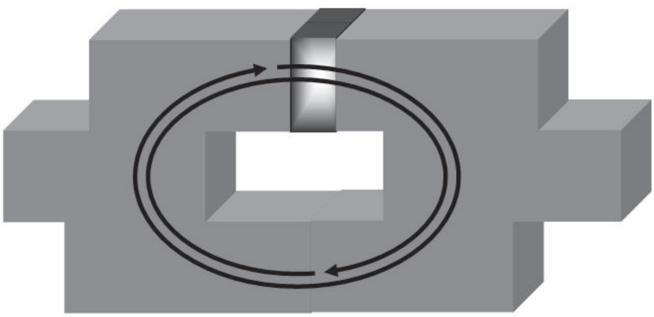


Figure V.12 If the magnetic field in the center of the superconducting ring is controlled to be exactly halfway between integral multiple of a fluxon, the superposition of the currents in two directions can be observed.

diffraction features (curve (a)) disappear if the optical grating is removed (curve(b)).

A remarkable experiment, which demonstrates direct quantum superposition, in a macroscopic complex system, is performed using the SQUID, which was discussed in the last chapter. In this case a superconducting ring with a single Josephson Junction is used. The magnetic field in the center of the loop can only take integral values of the flux quantum. Thus changes in the external magnetic field can be detected with a resolution of one fluxon or 2×10^{-15} weber. The reason for this quantization is quite interesting. As mentioned earlier, super currents flow in the superconducting ring to expel the magnetic field (Meisner Effect). The current is flowing in a region without an electric field (zero resistance). The wave function of the electron acquires an additional phase term as it circulates around the central field in the ring shown in Figure V.12. Since the wave function cannot have arbitrary values, the magnetic field in the centre of a ring has to be an integral multiple of the fluxon. If the external field is not an integral multiple of this value, a small super current flows in the ring and contributes an additional field so that the total field is an integral multiple. This current can obviously flow in either of two directions, one increases the field to the next higher multiple of the flux quantum while the other contributes an opposing field and decreases it to the lower integral multiple. If the external field is exactly half a fluxon larger than the integral multiple, the probability for either a clockwise or anti-clockwise flow of the current is equal. These are the two quantum states. Quantum mechanically, a superposition of these two states is possible. This does not represent half the electrons traveling in each direction but a strange state when all the electrons are moving in both directions simultaneously! Signature of this superposition state has been detected by careful experiments. The currents involved are of the range of microamperes, small by normal standards but definitely macroscopic. The system consists of about 10^{10} electrons. Controlled experiments have thus confirmed the applicability of the quantum ideas for complex macroscopic systems consisting of upto 10^{10} electrons. The process of decoherence, which will be discussed in a subsequent experiment, makes the life time of these

states extremely small. For example it is only a few nanoseconds in the present example. Obviously macroscopic objects have been demonstrated to exhibit quantum weirdness, though only for a very short interval of time and under stringent experimental constraints.

Quantum decoherence

The experimental difficulty of observing quantum superposition and duality, in macroscopic bodies is very obvious, since no object or event of human experience exhibits these properties. The transition of a macroscopic body from the quantum state (with the possibility of superposition) to the classical state (where superposition is not permitted) is normally attributed to decoherence. The environment, (which has also to be considered to be a quantum entity) gets "entangled" with the object being investigated. This means that, information about the object is rapidly disseminated into the surroundings. As a result, since (in principle) information such as the path followed by the object as it moves through one of the two slits is available, the interference, superposition etc. vanish. This entanglement is larger for macroscopic or complex systems and hence these quantum attributes are not normally experienced. In the past, this

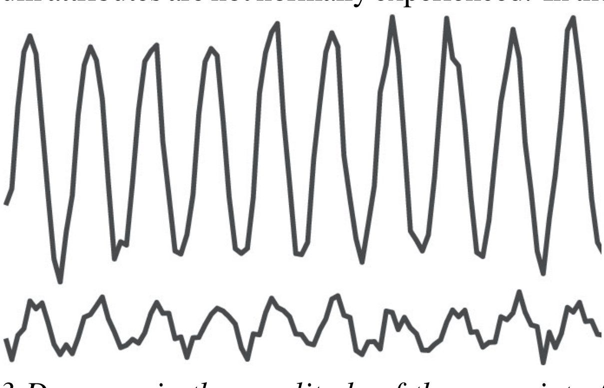


Figure V.13 Decrease in the amplitude of the wave interference of C_{70} molecules from (a) to (b) due to the addition of ~0.5×10^{-9} atmospheres of gas. (Adapted from K. Hornberger, S. Uttenthaler, B. Brezger, L. Hackermuller, M. Arndt, and A. Zeilinger, Collisional Decoherence Observed in Matter Wave Interferometry, arXiv:quant-ph/0303093 v1 14 Mar 2003)

had been a purely conceptual or thought process. Modern experiments have been able to demonstrate the slow decoherence due to interaction between the system and the "environment". Currently, the best example is the experimental observation of the decay of the interference pattern of matter waves of the C_{70} molecule. This experiment was performed using a series of gold diffraction gratings (rather than the optical grating used in the experiment described earlier) but the observation of interference between matter waves has been demonstrated not only for a spherically symmetric fullerene molecule but also for the biomolecule tetraphenylporphyrin ($C_{44}H_{30}N_4$ or "TPP") and the fluorinated buckyball $C_{60}F_{48}$. In a series of beautiful experiments, it was shown that the contrast between the fringes decreased exponentially with the presence of minute quantities of gas molecules in the system. Figure V.13 shows the decrease in the wave pattern, observed when gas was introduced to a pressure of $\sim 0.5\times 10^{-9}$ atmospheres in the experimental chamber. A collision with the lighter gas molecule cannot disturb the path of the heavy fullerene molecule but destroys the interference pattern because it carries sufficient information to determine the path that the interfering molecule has taken. This is similar to the complementarity experiment described earlier, where the path was determined using microwaves which cannot transfer momentum to the particles. The key difference is that the onset of decoherence is a continuous process and can be slowly observed. Controlled decoherence was also demonstrated for single quantum particles. In this case, a rubidium atom was specially prepared in a superposition state. This was then allowed to interact with a small number of photons present in a superconducting cavity. If the number of photons is zero, the superposition is not destroyed as the atoms travel in the resonator. But as the number is increased to 10, the superposition slowly decreases and is completely lost. Once again, the entanglement with the quanta is not accompanied by any momentum transfer. The mere presence of information that could potentially reveal the actual state of the system destroys quantum coherence.

Measurement without interaction

In the Heisenberg microscope, an attempt to determine the position of a quantum object, (such as the electron) results in an interaction, and

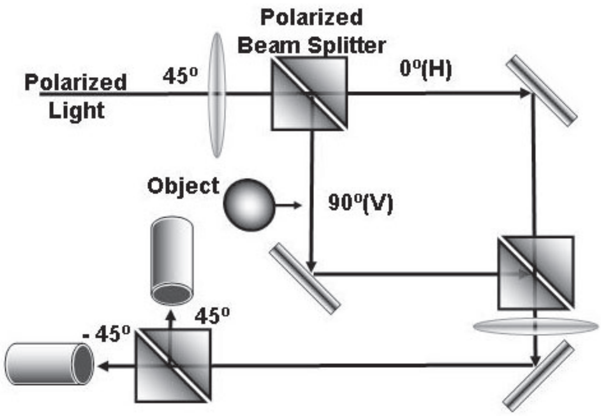


Figure V.14 Schematic setup that can detect the movement of the object into the path even though the interaction itself is not monitored.

consequently a change in the momentum. The description of this thought experiment points out the inevitability of interaction between the experimental setup and the system under investigation. The experiment being described here enables the detection of the presence of an object without any interaction with the object. The schematic is shown in Figure V.14. Light polarized at 45° is incident on an interferometer with two polarized beam splitters. The polarized beam splitter is a device which subdivides the incident light into vertical and horizontally polarized components which emerge from the device in different directions. The second of these devices acts in reverse, combining the two polarized beams. The third beam splitter is aligned so that the polarization of the incident beam is along one of the output beams (45° in Figure V.14). When there is no object, the light coming out of the interferometer is identical to the input. It is also polarized at the appropriate angle and only one of the detectors detects the light. When the object is placed in the path of the interferometer, there is no second beam of light at the second beam splitter for recombining. Thus the phase of the light emerging from the interferometer is not at 45º. The input to the third beam splitter is not identical to one of its outputs. Therefore, light is detected by both the detectors. There is a 25% probability of detecting the photon in one detector and 75% in the other. The photons actually reflected or absorbed by the object have not

been detected. In this scheme, while they are not actually detected, 50% of the photons incident on the first beam splitter are incident on the object. More complex approaches have been used, where the percentage is as low as 15%. Thought experiments have also been outlined, where the fraction is as low as 0%. These are purely quantum approaches unlike the above system which actually operates in the classical regime. This is a key demonstration of translating the idea of interaction from the thought experiment to the laboratory.

Detecting a non-observable quantity

In quantum theory, the probability is the product of the wave function and its complex conjugate. Addition of a phase to the wave function does not alter the probability. If a phase angle è is added to a wave function ø, one obtain a new wave function $øe^{iè}$. Thus the probability is unchanged since $øø^* = øe^{iè}ø^*e^{-iè}$ for all possible values of è. This is a continuous symmetry and consequently Noether's theorem predicts a conservation law. The consequence of phase symmetry is the law of conservation of charge. The phase therefore appears to be a mathematical entity without

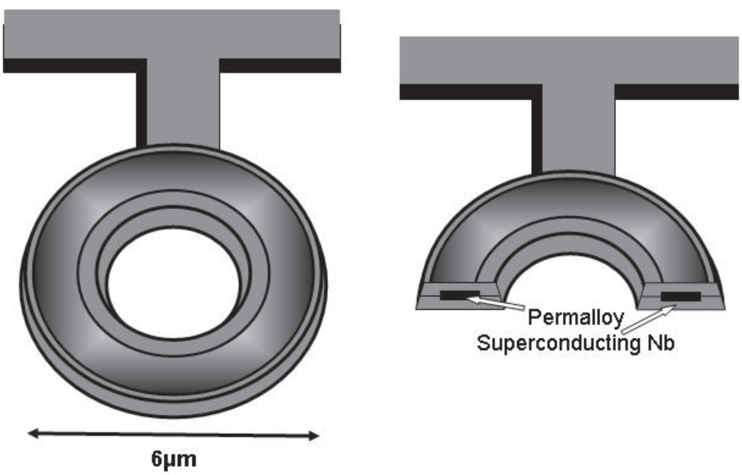


Figure V.15 The schematic view of a specially constructed toroidal magnet of 6 ì diameter. The toroid is covered by a layer of superconducting niobium as seen in the cutout. (Used with permission from Dr. Akira Tonomura)

an observable consequence. It should be noted that classical theory also introduces and uses similar entities. In classical electrodynamics, vector potentials are defined and used mathematically. However, it is emphasized that the value of the vector potential is not physically relevant. As mentioned earlier, flux quantization in superconducting rings is also a consequence of electron phase.

Aharonov and Bohm showed that the phase of the wave function of a moving electron is influenced by the presence of a magnetic field, even when the electron is excluded from the region of the magnetic field. This follows from the fact that the electron movement can alter the phase. Classically, the field being zero does not automatically ensure that the vector potential also be zero. This theoretical conjecture (thought experiment) was experimentally confirmed using a specially constructed toroidal magnet shown in Figure V.15.

A permalloy permanent magnet is made in the form of a toroid, 6 ì m in diameter and covered with a layer of superconducting niobium. As discussed earlier, a superconductor is a perfect diamagnet and super currents flow so as to prevent the penetration of magnetic fields. In view of the geometry, the magnetic field is completely confined and when the assembly

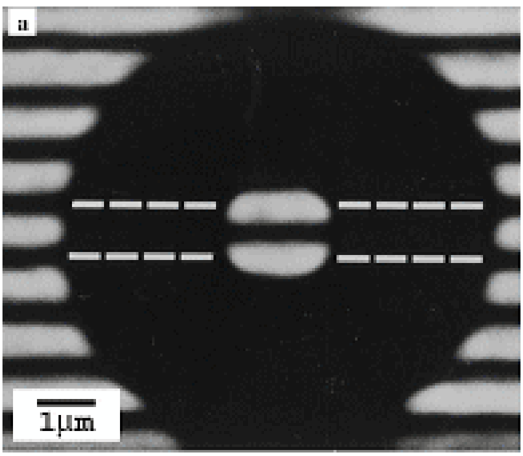


Figure V.16 Electron interference patterns obtained using the toroidal magnet. (Used with permission from Dr. Akira Tonomura)

is cooled with liquid helium, there is no magnetic field in the center of the toroid. Electrons are allowed to move both on the outside of the ring and through the narrow hole and form an interference pattern. Phase difference between the interference fringes formed by the electron beam passing though the hole in the toroid and passing on the outside is shown in Figure V.16. The fringes are displaced by just half a fringe-width inside, as compared with the outside, indicating the change of phase of electrons due to a field which they cannot actually experience. The force on the electrons due to the magnetic field is normally mediated by exchange of "virtual photons" (discussed next). Here, there is no exchange of virtual photons between the electrons and the magnetic field. The phase of the wave function of the electrons is still altered as predicted by quantum mechanics. Observation of the Aharonov-Bohm effect is another confirmation of the strange quantum phenomenon, observing the phase of the wave function which does not contribute to the probability of the quantum event.

Virtual photons and their detection

The Coulomb force (of attraction or repulsion) between two charges is proportional to the product of the magnitude of the respective charges and inversely to the square of the distance between them. This simple statement in science text books hides one basic question which fortunately for the teachers, students never ask. How is the presence of the second charge known to the first? In the more advanced books, an answer is provided by postulating the existence of an electric field around each charge. The charges then respond to the existence of the field, in the form of lines of force, and the consequence of this is equivalent to a Coulomb force. While the students are taught the methods to determine the magnitude of the field, the actual nature of the lines of force remains unclear. The situation with respect to the magnetic field is identical.

When the classical electromagnetic field theory is replaced by the quantum field theory, the charge instead of causing a field actually emits photons. These are then absorbed by the second charge leading to the observed Coulomb force. Since these photons are emitted and absorbed

without being detected, they are labeled as virtual photons. This process is called the exchange of a virtual photon. It is quite intriguing that the exchange of a photon leads to an attractive force between unlike charges and a repelling force between like charges. Exchange of virtual photons implies exchange of momentum. If we imagine this exchange to be classical, like two humans running along and exchanging a heavy ball, the two would move apart as if there is a force of repulsion. When the first person throws the ball, due to conservation of momentum, he moves away from the direction of the ball and when the second catches it, he moves in the direction along which the ball was moving for the same reason. However this analogy is fundamentally wrong, since the first charge, emitting a photon, cannot know the direction in which the second charge exists nor whether it is a like charge (for repulsion) or an unlike charge (for attraction). Also applying uncertainty principle to the virtual photon, the momentum being exactly defined, its position is completely uncertain and the direction of momentum is not defined in advance and thus the exchange of a quantum object is not confined to a repulsive force, as in the classical example. However, a mathematical analysis of the wave functions of the charges and the virtual photon is necessary for demonstrating the attractive force between unlike charges and repulsive force between like charges.

Not merely charges but even vacuum is expected to be filled with short lived virtual photons. This follows from the Heisenberg uncertainty relation between energy and time. If a photon of energy äE is created for a time interval ät and if äEät < h then the process is permitted by quantum theory. Thus vacuum must consist of a large number of virtual photons being created for short intervals of time. The energy and hence the wavelength of the virtual photons, depends on the location in which it occurs. Inside a cubic metallic box of size “a”, photons of wavelength larger than a/2 cannot be created. The reason is easier to visualize if the photon is considered as an electromagnetic wave. Then the electric field has to be zero at the surface of the metallic conductor. If the field is not zero, the electrons in the metal would move due to the field and cause resistive heating. Thus, the maximum wavelength permitted is twice the size of the box. It is just like plucking a string. The longest wave it can support while vibrating is the one where both ends are fixed. Similarly, if

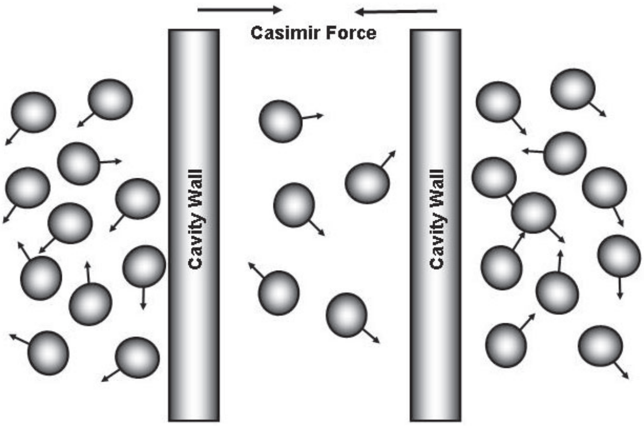


Figure V.17. Casimir force is caused by the presence of smaller number of virtual photons between the two conducting plates.

two parallel conductors are considered, the limitation mentioned above is valid in one direction but not in the other two directions where there is no metal. This is similar to the minimum wavelength of a phonon (vibration of a crystal lattice), described in Chapter III. There, the minimum wavelength caused atoms to move in opposite direction. The number of virtual photons created between the parallel plates is smaller than that created outside as schematically depicted in Figure V.17. Thus, more virtual photons impinge

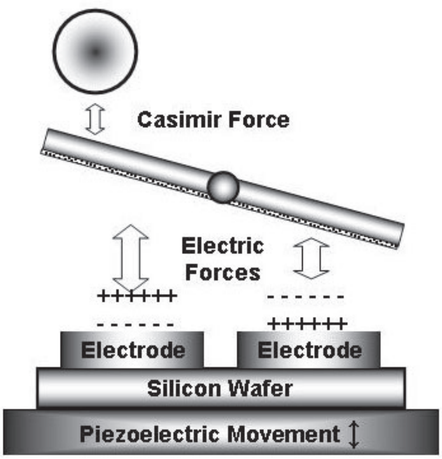


Figure V.18 Schematic of the micro machined setup for detecting Casimir force.

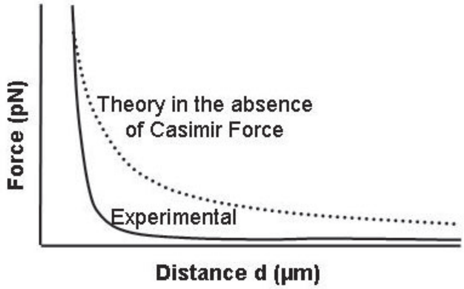


Figure V.19 The variation of the experimentally measured force between the sphere and the torsion rod confirms the existence of Casimir force.

on the plates from outside and transfer more momentum to the plates than from the inside. There should be a small force of attraction between the two plates purely because of the virtual photons. This force was theoretically predicted and called Casimir Force which varies inversely as the cube of the distance between the plates.

With the development of modern micro fabrication techniques it has become possible to measure this force experimentally. The experimental system used to determine the force between a sphere and a micron sized cantilever machined in silicon is shown in Figure V.18. The torsion rod is provided with a fulcrum to move like a see-saw. Small electrodes move the torsion rod by electrical attraction. Extra voltage is to be applied to contact on the left since it has to oppose the Casimir force of attraction between the torsion rod and the sphere. The experimentally determined variation of the force with the distance separating the two bodies is shown in Figure V.19 and confirms the mathematical calculations made for this specific geometry. Thus while the virtual photons are by definition those that have not been detected, exchange of virtual photons is a quantum process that results in other physical phenomena observed in the laboratory. The Aharonov-Bohm experiment described earlier detects the change in "phase" of the electron wave function ø. The phase does not alter the "probability of occurrence", the basic observable of a quantum process. Casimir Effect is the consequence of non-observed or "virtual photons". Thus, quantum theory is not "all probability alone", an issue that will be

discussed in the final chapter. The limitation on the creation of virtual photons in a cavity has another spectacular physical consequence as discussed next.

Controlling spontaneous emission

Bohr's extremely simple model of a hydrogen atom predicts with a very high accuracy the frequencies of the observed spectral lines of hydrogen. He showed that if the angular momentum of the electron considered as a classical particle moving around the proton is assumed to be quantized (an extension of the Planck's law), the differences in energy states are equal to observed lines in the hydrogen spectrum. The electron when excited to the higher energy state spontaneously returns to the lower energy state and emits the excess energy in the form of a photon. The time taken for the transition is once again related to the Heisenberg uncertainty. With time, the uncertainty in energy of the electron increases and in due course this becomes larger than the difference between the two energy levels. At that point, the probability of the electron being found in the lower energy state becomes large and the transition occurs. This is an average life time very much like the life time of radioactive elements. Only the probability of the occurrence for a large number of atoms can be specified and not the life time of a specific atom.

The time within which the emission takes place can be experimentally determined, in some cases. The experiment being presented

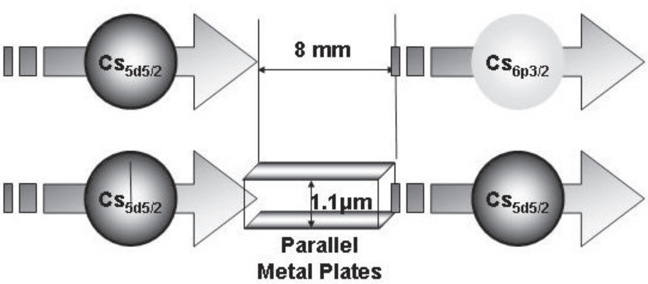


Figure V.20. The $5d_{5/2}$ ➔ $6p_{3/2}$ spontaneous emission is suppressed when the atoms are confined to the space between two parallel metal plates.

here involves the emission of a microwave photon of wavelength 3.49 microns by a Cs atom ($5d_{5/2} \rightarrow 6p_{3/2}$). A beam of Cs atoms are formed in an oven and move to a detector placed at a distance of 8mm as schematically shown in Figure V.20. Since the velocity is 400 m/s, the atoms takes 20 ì s to reach the detector. If the atoms are excited to the $5d_{5/2}$ state at the source, the probability of detecting an excited atom is about one part in a million. The life time is much smaller than 1 ìs. In the second experiment, the path between the source and the detector is confined between two metallic plates separated by a distance of 1.1 microns as schematically shown in Figure V.20. It is observed that at least two percent of the atoms do not decay before crossing the plates. This experiment shows that the spontaneous emission time has been increased by a factor of ~12.8.

This surprising result is understood when it is realized that a photon of 3.49 micron wavelength, which has to be emitted by the excited Cs atom cannot exist between metal plates separated by 1.1 microns. This is similar to the creation of virtual photons between parallel plates discussed earlier. The "quantum surprise" is that the excited atom is "aware" of the impossibility of emission without in any way being able to probe the environment and finding the metallic plates. This experiment indirectly confirms the discussion made above that spontaneous decay occurs when the uncertainty in the energy is larger than the separation between the two energy states. A more interesting aspect of this intricacy is revealed in the next experiment.

Quantum Zeno

The Zeno paradox discussed and debated in ancient Greece "proves" logically that "you can never catch up". This is illustrated with a race between the Greek hero Achilles and a tortoise which was given an initial handicap of running a shorter distance. The argument points out that it takes Achilles a finite time to reach the starting point of the tortoise. During this finite time the tortoise has moved some distance ahead. Achilles takes a further finite time to reach this new place. By this time the tortoise

has moved further and so on. Therefore, Achilles is forever trying to catch up and never succeeding. The Greeks discussed related paradoxes such as "you cannot even start" and "you cannot even move". All these paradoxes emerge out of the inability to sum infinite parts into which a finite time duration has been divided. The error is in assuming that the sum of infinite number of parts has to be infinite even when the parts are infinitesimally small. The paradox can be resolved using modern mathematical methods.

The Zeno paradox has become a source of inspiration for a peculiar quantum experiment labeled the "quantum zeno" which proves that "a pot that is watched does not boil". This experiment is performed using cooled Be ions confined in a magnetic trap, described in chapter III while discussing Bose-Einstein condensation. 5,000 beryllium ions are confined in an electromagnetic trap at a temperature less than 250 mK. Normally, when exposed to laser radiation of 313 nm, the Be ion is excited from state 1 to state 3 as shown in Figure V.21 while exposure to 320.7 MHz rf excites the Be ion to state 2. If the ion is in state 1, the laser light is scattered since it can be absorbed causing the (1➔3) transition and subsequently scattered in any other direction by undergoing the (3➔1) transition. In the experiment, a 256 ms pulse of 320.7 MHz rf is applied. Normally this is sufficient to transfer all the ions to state 2. The 313 nm laser pulses of duration 2.4 ms are used as probes. As described above

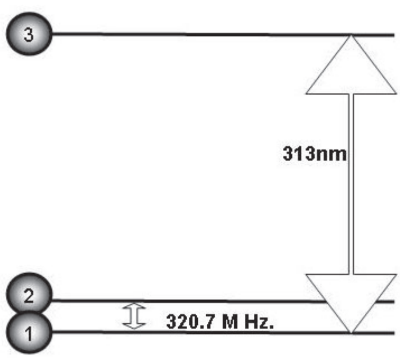


Figure V.21 Energy levels of the Be ions in the quantum zeno experiment.

scattering is an indication that the atoms are in state 1. If one laser pulse is applied after the application of the 256 ms rf pulse, all ions are already in state 2 due to absorption of the rf radiation and none in state 1 and there is no scattering. If more than one laser pulses are applied simultaneously with the 256 ms rf pulse, the number of ions in state 2 decreases and there is some amount of scattering. The amount of scattering increases with the number of laser pulses and when the number of pulses is 64, there are no ions in state 2. The measurement pulses of laser radiation (watching the pot) prevents the absorption of the rf energy (boiling) and hence the name quantum zeno. In quantum theory, the use of the laser pulse to probe the state of the ion, the measurement process, interrupts the evolution of the ion's uncertainty to become large enough to permit the 1➔2 transition despite the presence of the appropriate rf radiation. These experiments are selectively cited for philosophical discussions regarding the meaning of quantum mechanics, an issue that will be discussed in the final chapter.

Practical applications

The physics community is interested in experiments such as the Aharonov-Bohm, quantum zeno or entanglement in the spirit of a mountaineer. The only reason for climbing a mountain is because it is there. The philosophers are generally intrigued but are selective in choosing experiments for their discussions. At the same time these experiments have some very significant technological possibilities which will be discussed below. The quantum experiments described in this chapter are very recent.

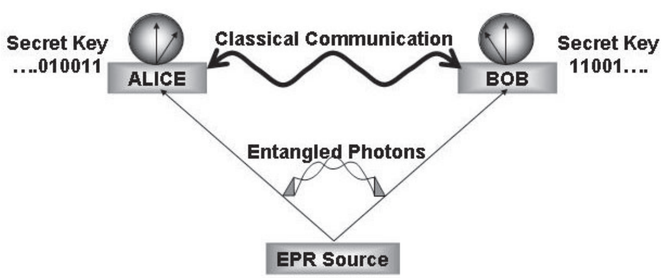


Figure V.22 Schematic of the distribution of quantum cryptographic key using entangled photons.

They are expected to increase the precision of measurement techniques. Squeezed light sources can be used for measurements below the quantum noise level and for low radiation level measurements on biological samples. Improved accuracy measurements are currently enabling the detection of minute changes in the values of universal constants over the lifetime of the universe. The quantum zeno phenomenon can be possibly used to develop a better standard of frequency than the atomic clocks being used today. Weak transitions with lower line width can be detected by temporarily "shelving" the electrons in the upper level of weak transition which prevents the absorption corresponding to the strong transition. Only two examples, one a commercial technology, "Quantum Key Distribution" (QKD) and the other a technological dream, "quantum computing" will be described below.

Quantum cryptography

The use of coding decreases the possibility that unauthorized persons can access the information. The distribution of a key, which can be used to code and decode messages, is a major problem. Also, the code should be unbreakable, that is to say the code should not be easily deduced by studying the coded message. The technology based on the quantum ideas discussed in this chapter, which has already emerged in the market place is Quantum Key Distribution. Two types of quantum approaches are now commercially available. The first uses the entangled photon pairs discussed in the EPR experiment described above and the second uses pulses of polarized light consisting of single photons.

In the first case, as shown in Figure V.22, the pair of entangled photons is distributed to two recipients who are always described in literature as Alice and Bob. The two measure the polarization in either "Horizontal /Vertical" or "+45° /-45°" orientations (measurement basis). A successful photon detection is identified as bit "1" and the absence as bit "0". The bit value, measurement basis, and the time of detection are recorded. The time of detection and basis is exchanged on the classical communication channel which need not be secure since this information does not enable the reconstruction of the code. Since the choice of the

basis is random with 50% probability, there will be coincident detections using the same basis. Since entangled photons are used, when measured using the same basis, the polarizations of the two photons are opposite to one another. These bits form the secret code which cannot be eavesdropped. Any such attempt will result in collapse of the wave function and will be reflected in the correlation statistics. Such schemes have been used at distances upto 10 km in free space and longer distances in fibers. Commercial, automated systems are presently available.

The second system uses polarized single photons. Once again the 4 states of polarization “Horizontal/Vertical” and “+45º/-45º” are employed. For example, “0” bit is sent on a fiber as a single photon with the polarization “Horizontal” or +45º” and bit “1” as “Vertical or “-45º. For each arriving photon, the receiver randomly selects among the two bases for measuring the polarization. He records the choice as also the result of the measurement. As in the case of the entangled photons, the lists of the orientations are exchanged and the coincident bases are selected. Figure V.23 shows a small string of bits as a sample. The shift key which is half the length of the total is used as the code. Eavesdropping results in significant changes in the statistics of coincidences and by comparing a small length of the string on the open channel, the integrity (absence of eavesdropping) can be confirmed. Commercial instruments for performing this key exchange are now available and used by the defense forces, commercial banks etc.

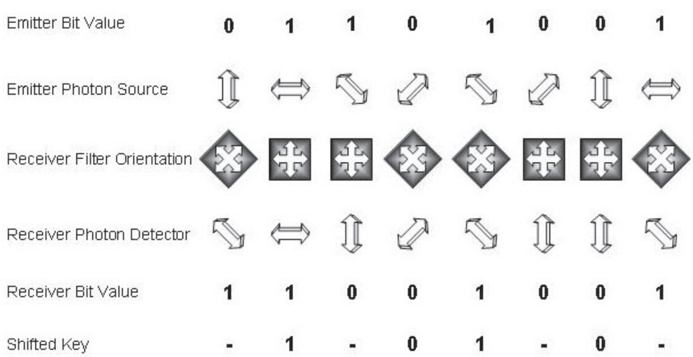


Figure V.23 Details of the single photon polarization measurements leading to a secure key.

Quantum computing

The improvement in performance of conventional computers is achieved by continuously decreasing the size of the electronic devices. It is anticipated that this decrease will be limited by quantum effects. At some stage, the collection of small number of atoms cannot be described by the band picture of semiconductors explained in Chapter III. In the resonant tunneling transistor described in Chapter IV, the states are atomic single energy levels rather than bands. There are efforts at developing molecular computers to address these concerns.

A more basic problem is the fact that all current computers, technically called von Neumann computers use Boolean Algebra. The bits have fixed values of "0" and "1" which correspond to logical statements "True" and "False". At the fundamental level, the entire computer program is the implementation of the logical propositions "AND", "OR" and "NOT" on these bits. An n bit number is thus represented by a string of n 0's and 1's.

Quantum computing is fundamentally different and uses a qubit or quantum bit which is a superposition of the states "1" and "0". Unlike classical bits, qubits can exist simultaneously as 0 and 1, with the probability for each state given by a numerical coefficient. Describing two-qubits requires four coefficients. In general, n qubits demand 2^n numbers. For example, if n equals 50, about 10^{15} (2^{50}) numbers are required to describe all the probabilities for all the possible states. A quantum computer promises to be immensely powerful because it can be in multiple states at once and act on all possible states simultaneously. Thus, a quantum computer could potentially perform a very large number of operations in parallel, using only a single processing unit. A quantum superposition involving N objects can in principle store all the numbers from 1 to N!-1 (N! is factorial N, the product of all numbers 1, 2, 3….N). The identification of quantum algorithms for specific jobs (factorization of numbers and data base search) superior to any classical algorithms has encouraged research in the area. At present, the research effort is concentrated on demonstrating

superposition states in practical devices. Superposition states have been realized with electrons, cooled ions, cooled atoms, Cooper Pairs, single implanted atoms etc. However, the only system where quantum computing has been demonstrated uses nuclear magnetic resonance (NMR).

Nuclear Magnetic Resonance (NMR) computer uses specially selected (sometimes specially produced) molecules of liquid as independent quantum memory registers. Qubits are implemented as spin states of the nuclei of atoms comprising the molecules. The liquid is subjected to a strong magnetic field (9 to 15 tesla) and we know that a magnetic moment is associated with the spin of the nucleus (protons and neutrons). The energy depends on whether the spin is aligned in the direction of the magnetic field or opposed to it. For each spin, there are quantized values. This is similar to the discussion of the Stern and Gerlach experiment. Spins can be moved from one state to another by using radio frequency radiation and by altering the direction of the magnetic field. As the nucleus interacts with the surrounding electron cloud and with other nuclei within the molecule, the energy levels change. By studying the frequencies at which radiation is absorbed, it is possible to infer the chemical composition and structure of a molecule. If the chemical structure is known, nuclear spins can be manipulated by appropriately tuned frequencies. The most powerful NMR computer demonstrated so far is a 5-qubit system built by Isaac Chuang and his colleagues from IBM Almaden Research Center, Stanford University and the University of Calgary in August 2000. The computer uses a molecule with five fluorine atoms designed to provide the required quantum interactions between nuclear spins. The computation, which solves the order-finding problem, comprises 200 logical steps (quantum gates), which were executed in about 0.3 s. Quantum computing is currently only an attractive research problem rather than a viable technological reality but the attempts show the close interaction between ideas of entanglement, decoherence, superposition etc. and the technological world.

The discussion in the present chapter showcases the ability to probe the quantum nature in minute detail. Ideas which at the time of the development of quantum theory were considered as "thought experiments" have now been implemented and the results quantified. The results seem

to be in agreement with Bohr's idea that quantum objects are too small for conscious perception and need not be described in intuitively obvious terms. One technological product that has emerged in the marketplace has also been described to emphasize the practical use of these ideas. However these experiments in some sense deepen the quantum mystery. A brief and personal summation of all the ideas in the book is presented next.

VI
The Summing Up

Quantum mechanical description requires the postulation or assumption that quantum entities with bizarre properties exist. These properties were then described and it was subsequently shown that experiments can be performed on the lightest material particle, the electron and on the particle of radiation, the photon to confirm the existence of quantum entities. It was further seen that most phenomena in macroscopic systems, whether the common observation of the transparency of glass or the esoteric superfluid state of liquid helium, that can only be observed in a precision experiment, can only be explained as the consequence of the collective properties of quantum particles. The technological use of quantum ideas was described followed by the latest of the quantum experiments probing the mysterious properties of quantum objects, experiments performed out of pure curiosity but potentially useful for new technologies.

The gaps in this presentation

Before an attempt is made to collate all these ideas and provide a summing up it is necessary to identify what has been left out. One omission that should be obvious even to a non-scientific reader is of nuclear particles namely quarks, gluons, neutrinos and so on. While aesthetically these appear to be more important, since they appear somehow to be more "fundamental", for the present discussion, they shed no further light on quantum ideas than the complexities of condensed matter. They have been extensively discussed in various popular accounts. While discussing the various experiments, examples have been cited to show that light is a

wave, that it is a particle, that is it both within the same experiment and that it is neither. It should be noticed that an experiment like "interference" which is simple to understand as a wave phenomenon, can be discussed as a collection of quantum particles. The best description of classical optics in terms of quantum particles is of course Feynman's QED. Feynman shows in his book, how all optical experiments, described using wave picture in text books can be described using particle picture also. In the present book, the particle or wave descriptions were selected as needed. Alternate descriptions are possible but were not discussed. Whether the theory is applicable at very small dimensions of the order of 10^{-35}m, (the Planck Scale) is beyond experiments. Speculative theories, which rarely draw verifiable conclusions are extremely popular but have been ignored here. In this as in most of the book, the comments of Feynman, treating "Superstring Theories" as applied mathematics have been quite influential. Finally arguments that human consciousness itself is a quantum phenomenon have not been discussed.

Is a "summing up" necessary?

It is not perhaps really necessary to offer personal concluding comments. One can simply accept the wise words of Bohr that quantum objects are too small for conscious perception and need not be described in intuitively obvious terms and that there is no need to be uncomfortable with the quantum strangeness. One can even join Feynman and claim that it is delightful to use absurd assumptions regarding the properties of quantum objects and find a mathematical theory that predicts experimental results with incredible accuracy. One can also mildly grumble that "maybe the old one is really malicious" as Einstein did shortly before his death. He had argued for over twenty years that quantum theory was incomplete and that "god was not malicious" and that "he does not play dice". Not very surprisingly the latter statements achieved more publicity.

After all, there does not seem to be any revolutionary change in our knowledge of the quantum nature since Feynman wrote his book which has been cited a number of times in this presentation; perhaps even since the famous "Bohr-Einstein" arguments or even earlier. In a remarkable

foresight, Einstein immediately identified Bohr's atomic model as introducing a new approach to physics with a physical theory that is limited "in principle". It provides only a limited description of nature. It is to be noted that this was some fifteen years before the uncertainty principle was formulated by Heisenberg. A famous physicist, Freeman Dyson has recently explored seriously, the possibility that we may never be able to resolve the dichotomy between a classical general theory of relativity and a quantum explanation of the rest.

But the worry regarding what quantum theory really means is quite widespread. Not even all physicists are able to simply accept the Feynman approach that since we can do good physics even without worrying about these matters, we can ignore it. One example of the unrest on the part of main stream physicists is reflected in the importance afforded to the wave function. When the experiments are described in terms of a wave function, it is necessary to accept a continuous evolution of the wave function followed by the collapse of the wave function after the measurement. With the new experiments reported in the last chapter, the perception grows that the wave function is somehow something real.

Since measurement itself has a major role in quantum physics, there are eternal arguments regarding, how much of the experimental setup has to be described quantum mechanically. If microscopic entities like atoms are quantum objects, so are the measuring systems a set of atoms. The issue of a conscious observer becomes apparent in certain descriptions of important quantum experiments. When is the act of measurement complete? Is it when the results are understood by a conscious observer? Serious physicists have argued this problem and made strange arguments ending with a claim that the universe itself is a quantum entity and that it came into existence when it was consciously perceived.

There has been some effort in main stream science literature in attempting to find alternates to the so called "Copenhagen Interpretation". The explanation of quantum mechanics, developed under the leadership of Neils Bohr and used by the entire physics community is given this name since it originated in Copenhagen. This gives a mistaken idea that

alternatives exist. Several equivalent mathematical formulations exist but these are not "interpretations". Statements, such as that by Peirles that there is no other interpretation are considered scientific (or physics) arrogance. These discussions and arguments have been extremely attractive to the non-physicists in making elaborate and philosophical discourses. Quite often the Copenhagen Interpretation is even called the non-interpretation (at least in philosophical circles).

"Fools venture where angels fear to tread". From now on, I will try to say something original and hence possibly foolish. Some comments on the scientific approach and the mathematics of quantum theories are being included here. These are not required for understanding the experiments as described in previous chapters but they are needed for the rest of the arguments being developed here.

The scientific approach

All human knowledge, as distinct from fiction depends on observations. This applies to science as also to religion and philosophy which also attempt to provide guidelines for human action. Observation of the reality of human desire, action, happiness and suffering is the basis for all religions and philosophies. Religion is fundamentally the notion that human action has non-physical consequences. Many aspects of life that could be observed but not understood were assumed to have non-physical causes. This made it almost axiomatic for all religions to expect a non-physical consequence for human action. This is the reason we have a religion that does not invoke a personal god, like Buddhism, but even here, there is non-physical consequence of human action called "Karma". More importantly, philosophy and religion use very few experimental data. Only very few examples or paradoxes are invoked. The discussions are very elaborate and counter-examples are neither cited nor used to modify the discussions. Consequently they have extremely limited capability of predicting the result of a future experiment and become dogmatic. Primitive science in all cultures shared many of these attributes. In the language of physicists the data are few and errors large, making science impossible.

Science as it developed from the time of Galileo is an exercise to mathematically correlate a large number of data in a quantitative way and to predict the experimental results before they are actually performed. A scientific result is valued based on its ability to perform these two functions – namely prediction and quantitative accuracy. Newton achieved his fame because of the ability of his theory to provide a mathematical description of both the movement of heavenly bodies and of cannon balls. Halley used the theory to predict that the comet being observed in 1705 would return after 76 years. This was confirmed after Halley's death and the comet was named after him. This is a small example of quantitative scientific prediction. No experimental result contradicted Newtonian gravity till the observation of the bending of star light by the sun during the total solar eclipse in 1919. Once this was observed, Einstein's general theory of relativity replaced Newtonian gravitation. The popular idea that Einstein proved Newton wrong is non-scientific. Einstein's theory has to agree with all the experimentally verified predictions of Newtonian theory. When the gravitation field is small, the two theories are identical. This is another important attribute of modern science. A new theory has to be identical to the existing theory and predict all experimentally verified conclusions of the older theory.

Mathematics of quantum theories

Quantum theory is an astoundingly successful mathematical theory. As mentioned earlier however, only the probability of the result of an experiment can be known. The result for an individual event is random. The probability of the experimental result can be calculated using many different mathematical models. This is not very much appreciated except by theoreticians. Each of these is in principle capable of determining the probability of occurrence of any given quantum event.

The most popular approach is the use of the wave function ø which is calculated from a knowledge of the physics of the system, basically the kinetic and potential energies of the various quantum particles. The wave function ø is a complex quantity that depends on the location in

space and time, while the real quantity øø* is the probability of occurrence of an event or result of an experiment. In this mathematical approach, ø evolves or changes continuously till the measurement is completed when it "collapses" into the final observable value of the probability. This description of the collapse of the wave function creates an illusion of the wave function being related to a "real physical entity". Actually however ø is only a mathematical quantity and is not observable. øø* is the only real quantity. A description in terms of ø has the major advantage that concepts such as superposition and entanglement become obvious. A common ø is defined for the two entangled particles and the collapse of the wave function results in the two observed particles with the specific properties.

The Feynman path integral formalism is the second most popular approach. It is very useful for understanding that a quantum particle does not travel in a specified trajectory. If the trajectory is actually defined, the uncertainty relation is violated. If the position (the path) is precisely known, the velocity is completely unknown and there is no meaning to the statement that the particle is traveling along the path. This approach considers the position and location of a quantum object, for example an electron and calculates the probability for it to be observed at a specific different location in space and time. All possible paths are considered and each path is assigned an amplitude which is a complex number. These paths may include some in which the electron briefly emits one or more photons and reabsorbs them. No constraints are placed on the paths. For example particles need not travel along straight lines. They may even have to travel with infinite velocity if a real particle has to travel the path. (The difference in position is large and in time small). The amplitudes for all the paths are all then added. The final complex amplitude is once again multiplied with the complex conjugate to obtain the experimental probability of observing the electron at the second location. This mathematics exposes the complexities in applying the special theory of relativity to quantum systems. Faster than light communication of signals is prevented by the definition of the amplitude and not by prohibiting the path. All paths contribute to the final amplitude and the overall probability. The calculated probability agrees with the experimental observations only if all paths are specifically in-

cluded. In spite of success, in this approach useful ideas such as the uncertainty relation, superposition and collapse of the wave function etc. become completely masked. The results obtained agree with the first method, namely use of wave function. The uncertainties in position and momentum are related as mentioned earlier but the point where these concepts are introduced into the mathematics is not obvious.

The easiest assumption regarding the strange properties of quantum objects is to assume the existence of hidden variables. The hidden variable theories assume that the probabilistic quantum theory has a substratum based on normal classical foundations. These are very popular particularly for those with an "objective" philosophical mind. Einstein was the first who desperately hoped to get a "deeper" understanding than that provided by the "quantum theory". However, it is now experimentally clear that any such theory must provide for instantaneous correlations, which to a large extent decreases their philosophical attraction. In actual scientific research on the other hand, there is no need for the hidden variables and Occam's razor can be invoked. This is a universal scientific practice of not using variables or concepts which are not necessary i.e., a theory with the smallest number of variables is always considered superior. Other theoretical descriptions exist and are used in narrow areas of advanced physics research.

As can be seen, one particular concept from among "non-observable wave function collapse", "lack of trajectory" or "instantaneous correlations" is more obvious in a specific mathematical description. No description exists without some concept which is not couter-intuitive in nature. In research, the choice of any specific mathematical description is primarily driven by its utility in attacking the specific scientific problems. For example, the hydrogen spectrum can be easily solved using the standard Schrödinger wave equation but was intractable until recently using the Feynman Path Integrals. This is not surprising. There is often more than one mathematical method to solve a specific problem. Some methods are much easier than the others and therefore preferred for that problem. The reverse may be true for some other problem. It is pertinent to note that

philosophical arguments always avoid the path integral approach since it really is not suitable for such discussions. There is no wave function that can provide an illusion of being a physical quantity. The evolution and collapse during measurement provide ample food for thought. In the path integral method, one explicitly starts to calculate the probability. This results in an immediate recognition that this is a mathematical procedure. This argument that the wave function approach is more "physical" is made not only by philosophers but also by some eminent physicists.

While considering the possibility of various equivalent mathematical descriptions, it is pertinent to note that a basic quantum description (calculating the probability) is not always the most useful in scientific understanding. During the initial development of quantum ideas, ad hoc quantization was employed quite successfully. For example an ad hoc quantization rule was used both by Max Planck and S. N. Bose to derive the equation governing black body radiation. Einstein quantized lattice vibrations to explain the variation of specific heats. Bohr's model of hydrogen atom is another example. These approaches continue to be used in teaching and research. There are also certain quasi-quantum descriptions which are very powerful and useful. In describing the physics of semiconductor devices, electrons and holes are considered as classical particles moving in a potential that is obtained as a consequence of the quantum interference effects. It is possible to describe an electromagnetic field as a collection of photons, and such a description is needed for a proper description of the laser. However, in describing the electromagnetic field in a radio antenna, a classical Maxwell's equation is more useful for both teaching and research.

The summing up

In philosophy, while arguing the consequences of one experiment or paradox, the others are completely ignored. Secondly the arguments are never allowed to break on the basis of a single negation as unviable. Finally, Occam's razor, the principle that the simplest theory is to be preferred is never invoked. For example, Kuhn's structure of "scientific revolutions" is a very elaborate discussion seriously discussed by

philosophers of science. He observed that a revolutionary concept in science is not immediately and universally accepted. The simplest conclusion is that the revolutionary new scientific idea becomes popular once there are researchers familiar with it and confident of using it for their own creative output. Kuhn was not describing a characteristic of science but of scientists. This conclusion is the simplest but is most unlikely to be accepted by philosophers who seem almost to prefer complexity for its own sake. On the other hand, the attempt here is to obtain a scientific conclusion keeping in view the principles mentioned above.

The experiments referred to in this book provide a deep appreciation of the mystery of the quantum phenomenon. ***All*** these have to be included in a coherent picture. In science all the experiments will have to be considered together. When there are many mathematically equivalent approaches for explaining a given physical phenomenon, the differences should not be a basis for philosophical arguments. The choice is based on a perception of ease of communicating the idea to students and the utility of the approach for uncovering interesting consequences in research.

Collapse of a wave function is not reflected in the path integral formalism. However, many serious physicists continue to see a distinction between the path integral formalism which is seen as "purely mathematical" while the wave function description is not. What such hair-splitting and word associations fail to recognize is that each mathematical description emphasizes one bizarre aspect. In the case of the path integral formalism, the multiple paths of a particle are clearly seen. This aspect of the reality is lost in the wave function description.

The question as to why the world at large is classical is not very profound. Classical world is experimentally true and the quantum theory is expected to provide the right answers in the classical limit. All results which are "explainable" on classical framework are also explainable on a quantum basis. The reverse is obviously not true. It is entirely possible to begin the discussion with experimentally observed condensed matter phenomena, and demand a coherent physical picture. The quantum theory

would be the result. There is really no justification for assuming that the so-called fundamental particles are in any experimental or theoretical sense different from the various quasi-particles encountered in discussing condensed matter phenomena. As mentioned before, "fundamental elementary particles" (quarks etc.) can be described as excitations following a broken symmetry just as phonons or Cooper Pairs. These quasi-particles exhibit the same quantum nature as the so-called fundamental particles. A combination of a quantized lattice vibration and two individual electrons, all of them individually quantum particles, together behave like a new quantum object exhibiting the entire range of quantum properties. An atom of helium consisting of six fermions in three dimensions behaves like a boson. The conversion of fermions into bosons in two dimensions is different. But the quantum behaviour in two dimensions is exactly as "real" as in three dimensions. Practical devices, which exploit the properties of electrons in two dimensions, are commercially available. Many more interesting quantum objects are likely to be hypothesized and experimentally confirmed.

Practical experiments of macroscopic objects showing quantum delocalization and superposition continue to prove the validity of the quantum formalism and while arguments about the conclusions of EPR experiments being "foolproof" continue, technology has quietly entered the marketplace. Recent experiments on decoherence make one very positive that the transition to the classical will soon be experimentally definable. Similarly, while it is quite interesting to argue about the strange quantum property called "spin" and how it has no value except when measured, practical devices have been designed, based on the use of spin and are extensively employed in each and every hard disk drive of the personal computer.

The role of consciousness in quantum measurement is another issue that depends on experiments being selectively cited. The measurement processes (watching the pot) prevents the absorption of energy (boiling) in the quantum zeno experiment; a popular example. Once again, the description of the intricacies of the Schrödinger's cats or kittens always makes a mystery of the quantum measurement.

On the other hand, consider the quantum eraser experiments. Interference is absent when the information is in principle known. The interference is lost even when the fullerene molecules are bombarded with gas molecules. Not because they are deflected from their path. They are not! But because the information is knowable, not necessarily to conscious observer, the interference is lost.

Consider as another example, the Casimir effect. This is an experiment that detects the consequence of the presence of virtual photons. The emission and momentum transfer of virtual photons is real and experimentally measurable. The emission and absorption for a virtual photon is no different from for a real photon and is a quantum event. The Aharonov-Bohm experiment detects the change in "phase" of the electron wave function ø. The phase does not alter the "probability of occurrence", the basic observable of a quantum process. Thus, quantum theory is not "all probability" alone.

But if all this is accepted, at least in some quantum experiments, a conscious observer is not needed. Then is it not likely that the necessity for a conscious observer in certain cases is purely a consequence of the description being employed? Just like the choice of path integral formalism eliminates the need for the uncertainty relation and collapse of the wave function?

Unfortunately many experiments cannot be easily described using several different formalisms. Even the hydrogen atom can only be solved with difficulty employing the path integral formalism and this was also achieved many years after the formalism was developed. In many cases the professional scientist gets no sense of achievement in analyzing the experiment using a second formalism unless it aids in research or teaching.

As usual it is only Feynman who does not fall a prey to speculations regarding the roles of mind or psychology in measurement. His answer to the problem of measurement has all the beauty of Einstein's observer traveling with a wave, but has not somehow received the exposure that it warrants. According to this description, there is no absolute definition of

measurement (as with many other things in life), but measurement becomes easier to understand when the accuracy with which a thing has to be measured is defined *a priori.* Then those properties of the individual quantum object can be measured with finite experimental probability, which can be correlated with an unlimited number of atoms.

Normally, the quantum object being observed, for example an atom and the experimental setup consisting of atoms are mentioned without quantification. Feynman's description is a refreshing contrast. A measurement is to be seen as series of increases in the number of atoms involved, starting with the single quantum object and ending when the number of atoms is sufficiently large. The number is sufficiently large and the measurement is complete when the number can be extrapolated to infinity. The spot on the photographic plate represents the property of the atom since it can be enlarged, projected on the screen, effect large vats of chemicals etc. It can be made to effect ever-increasing sizes of things, not because it is recognized in the brain of a conscious observer. A better appreciation of this approach could perhaps lower the importance attributed to consciousness in quantum measurement.

The experimental support to the mathematical description of quantum mechanics is really mind boggling. The means of understanding why a film of oil on water leads to colours of thin films or why salt dissolves in water or why a piece of glass is transparent or why a piece of iron is magnetic requires one to assume the existence of strange quantum entities. The human brain is able to conceptualize such entities and the mathematics required to explain the phenomena, using processes which themselves are quantum mechanical. We understand science in a chemical computer employing biochemical reactions which obey quantum rules. Unfortunately, the brain seems incapable of conceptualizing the quantum entities. The experiments described here make it quite clear that we have to live with this limitation and there is no escape from this limitation. Maybe this is the ultimate uncertainty. The decrease in uncertainty of prediction of experimental results achieved with quantum theory has unfortunately enhanced the uncertainty in understanding what it means.

Glossary of some Technical Terms

In the text, most technical terms have been explained when they are introduced. However, some terms have been defined precisely below for additional clarity

Bandgap	The range of energies which are not possessed by electrons in a solid.
Bosons	Quantum objects which have a symmetry of 2ð and are capable of condensation into a single state.
Broken symmetry	A change in system corresponding to a lack of isotropy in a physical property.
Coherence length	The length of a wave which retains a perfect phase relationship.
Coherence time	The duration of time over which the wave maintains a phase relationship.
Complex conjugate	A mathematical quantity obtained by changing the sign of the imaginary part of a complex number.
Complementarity	The principle that wave and particle properties are not exhibited in the same experiment.
Condensed bosons	The quantum mechanical state when all bosons are in a single energy state.
Cooper pair	A composite quantum particle formed by the joining of two electrons and a phonon.
Covalent bond	Bond formed between atoms when the individual electrons of two atoms are shared by them.

Decoherence	The transition from a coherent quantum mechanical system to a classical system without interference effects.
Diffraction	Transfer of a small amount of energy in the wave into a different direction due to an obstacle whose dimension is comparable with the wavelength
Diffraction limit	The limit on the minimum size of an object that can be observed due to diffraction of scattered waves.
Duality	The property of quantum objects to appear to be waves under certain experimental conditions and particles in others.
Electron	The smallest negatively charged constituent of matter outside the nucleus of an atom which controls the binding of atoms to form molecules.
Electron density	The number of electrons existing in unit volume.
Entanglement	Multiple quantum objects with a common wave function and constraints on the properties of individual constituents.
Fermions	Quantum objects which have a symmetry of 4ð and obey Pauli exclusion principle.
Ferromagnetism	The interaction between atoms (or electrons) responsible for exhibiting the magnetic property of orienting with the earth's magnetic field.
Flux quantum.	The smallest quantum unit of magnetic flux.
Fourier analysis	The method of decomposing a required signal into the sum of a sinusoidal signals with multiples of a base frequency..
Hall voltage	Voltage generated perpendicular to the direction of flow of current due to the presence of a magnetic field.
Hologram	A record of the interference pattern generated by interference of scattered coherent radiation that permits reconstruction of a three dimensional image of the object.

Hybridization	Mixing of orbitals of electrons of approximately equal energy when they are located near each other is space.
Interference	The process whereby energy from one part of the wave is transferred to another part due to interaction with a second wave.
Ionic bond	Bond formed between atoms where there is almost complete transfer of electronic charge from one atom to another.
Laser	Light amplification by stimulated emission of radiation leading to a high energy state of condensed photons.
Line spectrum	The precisely defined colour or wavelength of light emitted or absorbed by atoms.
Mean free path	The average distance traveled by a particle between successive collisions.
Orbitals	The region in space where an electron can be confirmed to exist.
Phonons	The quantum mechanical object representing the smallest energy of a vibrating bond between atoms in a solid, also called a lattice vibration.
Photon	The smallest individual constituent of energy or radiation.
Polarization	The direction of spin of a photon.
Polaron	A quantum particle that forms due to the interaction between the polarization of a conducting polymer chain and the mobile charges.
Quantum Hall Voltage	Voltage generated perpendicular to the direction of flow of current in a two dimensional electron gas due to the presence of a magnetic field which is a multiple of the universal constant h/e^2
Quantum mechanics	The theory used to predict the results of experiments involving the smallest parts of matter (atoms and sub-atomic particles) and of light (photons).
Reciprocal space	A mathematical construction that helps in visualizing the Fourier (sinusoidal) components

	of electron density which correspond to the intensities of diffracted energy.
Resonance	Mixing of molecular orbitals of approximately equal energy when they are located near each other is space.
Spin	The quantum mechanical property that distinguishes bosons and fermions.
Statistical mechanics	The theory of determining the properties such as temperature and pressure in terms of the energies of a large number of individual particles.
Superconductivity	The interaction between electrons that permits them to move in a solid without loosing energy and thus exhibiting zero resistance.
Superfluidity	The interaction between atomic bosons that permits them to condense into a single state leading to lack of macroscopic viscosity.
Superluminal signals	The signals that travel with velocity larger than that of light in vacuum.
Transistor	A semiconducting device that permits amplification of a current and thus the most important component of modern electronic devices.
Tunneling	The quantum mechanical process of movement through a region of higher potential energy (barrier) without actual increase of particle energy which could enable it to go over the barrier.
Uncertainty principle	The manifestation of quantum nature whereby experimental determination of energy or momentum to a degree of accuracy limits the accuracy with which time or position can be determined.
Wave function	The complex mathematical quantity that defines the physical properties of the system.
Zero point energy	The minimum energy of quantum mechanical oscillator which is needed to avoid the loss of uncertainty.

Suggestions for Further Reading

The book is a mixture of introduction to quantum mechanics, condensed matter physics and technology. The text has been designed both for the general public and to serve as a companion to undergraduate students studying physics. No suggestions for further reading are necessary for the latter audience. The standard course material that they study would serve.

Numerous books have been written introducing quantum mechanics to the general public. George Gamow's "Mr. Tompkins in Wonderland" and "Mr. Tompkins Explores the Atom" (Macmillan 1946, Cambridge University Press 1993) are among my favorites. Robert Gilmore makes accessible some complex concepts in "Alice in Quantumland" (Springer-Verlag 1995). Unfortunately there is no dearth of books linking quantum physics to mysticism and eastern philosophy; "Dancing Wu Li Masters" by Gary Zukav (William Morrow and Company 1979, Harper 2001) and "The Tao of Physics" (Shambhala 1975, 2000) by Fritjof Capra being slightly older ones. Such books provide a competent description of the physics but the underlying agenda has to be resisted. Other books such as "In Search of Schrödinger's Cat: Quantum Physics And Reality" (Bantam 1984) and "Schrödinger's Kittens and the Search for Reality" (Little Brown & CO. 1995) by John Gribbin provide insights into the latest quantum experiments and generally cover the material discussed in Chapter II. "Clues to Quantum Nature" and Chapter V "Performing "Thought" Experiments". Other books that cover the same area of quantum physics and its interpretation include "Entanglement"(Four Walls 1980) by Amir Aczel, "The God Effect" (St. Martin's Press 2006)

by Brian Clegg, "Introducing Quantum Theory" (Totem Books 2004) by J.P. McEvoy and Oscar Zarate, "Quantum Physics: A Beginner's Guide" (Oneworld 2006) by Alistair I. M. Rae and "The Quantum Zoo" (Joseph Henry Press 2006) by Marcus Chown

I am not aware of any book for general audience that covers quantum theory of condensed matter. Books on topics such as superconductivity, nanomaterials, lasers and semiconductor devices provide some information. Among the possible sources are "The Wonder Chip"(National Book Trust, 1998) by K D Pavate, "The Quantum Universe" (Cambridge University Press, 1987) by Tony Hey and Patrick Walters, "Lasers"(Facts On File 2006) by Charlene W Billings and Sean M Grady, "Superconductivity Today" (Wiley Eastern 1993) by T.V. Ramakrishnan and C.N.R. Rao, the old but wonderful book "Introduction to Superconductivity" (Pergamon 1969) by A C Rose-Inness and E H Rhoderick, and "The Quest for the Quantum Computer" (Simon & Schuster 2001) by Julian Brown. Some of these are not designed for the general audience but can be read without the necessity of prior physics education.

Incomparably the greatest book is however "QED: The Strange Theory of Light and Matter"(Princeton University Press 1985) by Richard Feynman. Thanks to the genius of Feynman, a general reader will find something new in every reading of what appears to at first reading to be very elementary. An in depth reading of this book will serve better than any other single source material in providing an understanding of quantum mechanics and more importantly, an ability to identify limitations of indiscriminate philosophical claims.

Index

Dr S T Lakshmikumar obtained a Ph. D. degree from Indian Institute of Science, Bangalore joined the National Physical Laboratory, New Delhi as a scientist in the materials division. He has been a visiting scientist at the Ransellaer Polytechnic Institute, Troy, USA as a C V Raman Fellow in 1992. His research interests include amorphous, photovoltaic, dielectrics and nanostructured materials. He is deeply interested in communicating advanced scientific ideas to the students and general public and has written an earlier book, "The quest for new materials" which was very well received. Here he draws on his understanding of physics to provide a basic experiment based understanding of quantum mechanics without mathematics.

Quantum mechanics is fascinating. It explains most physical phenomena with incredible accuracy. Also, the assumptions are so counter intuitive. Aside from these difficulties in conveying the essence to the students, philosophical ramblings of non-physicists are a source of confusion and chaos. This book explains, through a series of experiments, the strange properties of quantum objects. The book shows that a wide variety of physical phenomena, can only explained on a quantum basis and these ideas have resulted in many technological devices ranging from transistors and lasers to SQUIDS and quantum codes. At the end, it hopes to highlight the limited scope for interpretations and philosophies.

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